Compito 12.10.21

~~S1~~ - ~~S2~~ - ~~S3~~

ES1 P1278

a. $\int \frac{1}{x^4}\,dx = \int x^{-4}\,dx = \frac{x^{-4+1}}{-4+1} = \frac{x^{-3}}{-3} = \boxed{\frac{-1}{3x^3} + c}$

b. $\int \frac{5}{4}\sqrt[4]{x}\,dx = \frac{5}{4}\cdot\int \sqrt[4]{x}\,dx = \frac{5}{4}\cdot\frac{x^{1/4+1}}{\frac{1}{4}+1} = \frac{\cancel{5}}{\cancel{4}}\,x^{\frac{5}{4}}\cdot\frac{\cancel{4}}{\cancel{5}} = x^{5/4}$

$= \sqrt[4]{x^5} = \boxed{x\sqrt[4]{x} + c}$

c. $\int x^8\,dx = \frac{x^{8+1}}{8+1} + c = \boxed{\frac{x^9}{9} + c}$

ES2

a. $\int \frac{1}{2}x^{-3}\,dx = \frac{1}{2}\cdot\frac{x^{-3+1}}{-3+1} = \frac{1}{2}\cdot\frac{x^{-2}}{-2} = \boxed{\frac{-1}{4x^2} + c}$

b. $\int \frac{4}{x}\,dx = \int 4\cdot\frac{1}{x}\,dx = 4\cdot\ln|x| = \boxed{4\ln|x| + c}$

c. $\int -\frac{2}{3}x^6\,dx = -\frac{2}{3}\,\frac{x^{6+1}}{6+1} = -\frac{2}{3}\,\frac{x^7}{7} = \boxed{-\frac{2x^7}{21} + c}$

ES3

a. $\int \frac{3}{\sqrt[3]{x^2}}\,dx = \frac{3\cdot x^{-2/3+1}}{-\frac{2}{3}+1} = 3\,x^{\frac{1}{3}}\cdot 3 = \boxed{9\sqrt[3]{x} + c}$

b. $\int 8\sqrt{x}\,dx = 8\,\frac{x^{1/2+1}}{\frac{1}{2}+1} = \frac{8x^{3/2}}{\frac{3}{2}} = 8x\sqrt[2]{x}\cdot\frac{2}{3} = \boxed{\frac{16x\sqrt[2]{x} + c}{3}}$

c. $\int -\frac{1}{x^4}\,dx = \int -x^{-4}\,dx = -1\cdot\frac{x^{-4+1}}{-4+1} = \frac{-1\cdot x^{-3}}{-3} = \boxed{\frac{1}{3x^3} + c}$

In classe 12.10.21

ES5

A $$\int \frac{\sqrt{x}}{\sqrt[4]{x}}\,dx = \int \left(x^{1/2} : x^{1/4}\right) dx = \int x^{\frac{1}{4}}\,dx = \frac{x^{5/4}}{\frac{5}{4}} + K = \boxed{\frac{4}{5}\,x\sqrt[4]{x} + K}$$

B $$\int \sqrt{x\sqrt{2x}}\,dx = \int \sqrt[4]{2x^3}\,dx = \sqrt[4]{2}\cdot\int x^{3/4}\,dx = \sqrt[4]{2}\,\frac{x^{7/4}}{\frac{7}{4}} + K =$$

$$\sqrt[4]{2x^3} = \sqrt[4]{2}\cdot\sqrt[4]{x^3}$$

$$= \frac{4}{7}\sqrt[4]{2}\cdot x\sqrt[4]{x^3} = \boxed{\frac{4}{7}\,x\sqrt[4]{2x^3} + K}$$

$$(2)^{1/4}\cdot(x^3)^{1/4}$$

C $$\int x^3\sqrt{x}\,dx = \int x^3\cdot x^{1/2}\,dx = \int x^{7/2}\,dx = \frac{x^{\frac{7}{2}+1}}{\frac{7}{2}+1} + K = \frac{x^{\frac{9}{2}}}{\frac{9}{2}} + K = \frac{2}{9}\sqrt[2]{x^9}$$

$$= \boxed{\frac{2}{9}\,x^4\sqrt{x} + K}$$

E59

$$\int (x+\sqrt{x})\,dx = \int x\,dx + \int x^{1/2}\,dx$$

$$= \frac{x^2}{2} + \frac{x^{3/2}}{\frac{3}{2}} + K = \boxed{\frac{x^2}{2} + \frac{2}{3}\,x\sqrt{x} + K}$$

E66

$$\int \left(\frac{5}{x^4} - \frac{4}{x^3} + \frac{3}{x^2}\right) dx = 5\cdot\int x^{-4}dx - 4\cdot\int x^{-3}dx + 3\int x^{-2}dx$$

$$= 5\cdot\frac{x^{-3}}{-3} - 4\cdot\frac{x^{-2}}{-2} + 3\cdot\frac{x^{-1}}{-1} = \boxed{\frac{-5}{3x^3} + \frac{2}{x^2} - \frac{3}{x} + K}$$

E77

$$\int (x^2-(2-x)^2)dx = \int (\cancel{x^2}-4-\cancel{x^2}+4x)dx = \int -4dx + \int 4x\,dx =$$

$$= -4\int dx + 4\int x\,dx = -4x + \frac{\cancel{4}x^2}{\cancel{2}} + K = \boxed{2x^2-4x+K}$$

E80

$$\int \frac{x+5}{x}dx = \int dx + \int 5(x)^{-1}dx = \int dx + 5\int x^{-1}dx = \boxed{x+5\ln|x|+K}$$

E93

$$\int \frac{x^2-9}{x+3}dx = \int \frac{\cancel{(x+3)}(x-3)}{\cancel{(x+3)}}dx = \int x-3\,dx = -3\int dx + \int x\,dx = \boxed{-3x+\frac{x^2}{2}+C}$$

Compito 14.10.21

~~56~~ - ~~63~~ - ~~71~~ - ~~73~~ - ~~75~~ - 81 - 92

E56 P1278

A. $$\int (x-5)\,dx = \int x\,dx - 5\int dx = \boxed{\frac{x^2}{2} - 5x + c}$$

B. $$\int \frac{3\cdot x^{1/3}}{x}\,dx = \int 3\,x^{1/3} : x^{1}\,dx = \int 3\,x^{-2/3}\,dx = 3\cdot\frac{x^{1/3}}{\frac{1}{3}} = \boxed{9\sqrt[3]{x} + c}$$

C. $$\int \frac{1}{4}\cdot x^{-1/2}\,dx = \frac{1}{4}\,x^{1/2}\cdot 2 = \boxed{\frac{\sqrt{x}}{2} + c}$$

E63

$$\int (4x^4 - 2x^2 + 5)\,dx = \boxed{\frac{4x^5}{5} - \frac{2x^3}{3} + 5x + c}$$

E71

$$\int \left(\frac{1}{x^2} - \sqrt{x} + \frac{6}{x}\right)dx = x^{-2} - x^{1/2} + 6\cdot\frac{1}{x}\,dx = -\frac{1}{x} - \frac{x^{3/2}}{\frac{3}{2}} + 6\cdot\ln|x| + c$$

$$= \boxed{-\frac{1}{x} - \frac{2}{3}\,x\sqrt{x} + 6\ln|x| + c}$$

E73

$$\int (x^{1/2} + x^{1/3})\,dx = \int x^{1/2}\,dx + \int x^{1/3}\,dx = \frac{2}{3}\,x^{3/2} + \frac{3}{4}\,x^{4/3} = \boxed{\frac{2}{3}\,x\sqrt{x} + \frac{3}{4}\,x\sqrt[3]{x} + c}$$

E75

$$\int (x+1)^2\,dx = \int (x^2+2x+1)\,dx = \frac{x^3}{3} + \frac{\cancel{2}x^2}{\cancel{2}} + x = \boxed{\frac{x^3}{3} + x^2 + x + c}$$

E81

$$\int \frac{4x + x^2}{x}\,dx = \int 4\,dx + \int x\,dx = \boxed{4x + \frac{x}{2} + c}$$

E92

$$\int \frac{(x-1)(x+2)}{x}\,dx = \int \frac{x^2 + 2x - x - 2}{x}\,dx = \int \frac{x^2 + x - 2}{x}\,dx =$$

$$\int \left(x + 1 - 2\cdot\frac{1}{x}\right)dx = \boxed{\frac{x^2}{2} + x - 2\ln|x| + c}$$

In classe

E78 P1279

$y = \frac{1}{2}x - 1 \quad \leftarrow f(x)$

grafico $F(x) = ?$ quale rappresenta una primitiva di $y = \frac{1}{2}x - 1$

$$\int \left(\frac{1}{2}x - 1 \right) dx = F(x) = \frac{1}{2} \cdot \frac{x^2}{2} - x = \boxed{\frac{x^2}{4} - x + C}$$

studio il crescere/decrescere della derivata:

$\frac{1}{2}x - 1 > 0 \rightarrow x > 2$

	2	
$-$		$+$
$\searrow$		$\nearrow$

la risposta giusta e' $\boxed{C}$

Ricorda

Vertice parabola: $\left(\frac{-b}{2a} ; \frac{-\Delta}{4a} \right)$

E96

trovare primitive $F(x)$ di $f(x) = x^2 - \frac{1}{x}$, trova quella tale che $F(1) = 2$

$$F(x) = \int \left(x^2 - \frac{1}{x} \right) dx = \frac{x^3}{3} - \ln|x| + K$$

$F(1) = 2$

$2 = \frac{1}{3} - \overset{0}{\ln|1|} + K$

$2 - \frac{1}{3} = K$

$K = \frac{5}{3}$

$$\boxed{F(x) = \frac{x^3}{3} - \ln|x| + \frac{5}{3}}$$

E99

$P'(x) = \frac{x}{5} - 30$

x = numero biglietti venduti

$p(x)$ = profitto in €

$p'(x)$ = profitto marginale = rapidità di variazione

$K = 50$ € = guadagno fisso

$x = 500$ biglietti

$P(500) = ?$

$$P(x) = \int \left(\frac{x}{5} - 30 \right) dx = \frac{x^2}{10} - 30x + K = \frac{x^2}{10} - 30x + 50$$

$$P(500) = \frac{(500)^2}{10} - 30 \cdot (500) + 50 =$$

$$= \frac{5^2 \cdot 10^{\cancel{4}\,3}}{\cancel{10}} - 30 \cdot 5 \cdot 10^2 + 50$$

$(500)^2 = (5 \cdot 100)^2 = (5 \cdot 10^2)^2 = 5^2 \cdot 10^4$

$$= 25 \cdot 10^3 - 150 \cdot 10^2 + 50$$

$$= 25 \cdot 10^3 - 15 \cdot 10^3 + 50$$

$$= 10 \cdot 10^3 + 50$$

$$= \boxed{10'050 \text{ €}}$$

E103

$$\int (2e^x+1)\,dx = 2\int e^x\,dx + 1\int dx = \boxed{2e^x + x + c}$$

E107

$$\int (x+7\cdot 7^x)\,dx = \boxed{\frac{x^2}{2} + \frac{7\cdot 7^x}{\ln(7)} + c}$$

E110

$$\int \frac{9^x}{7^x}\,dx = \int \left(\frac{9}{7}\right)^x dx = \boxed{\frac{\left(\frac{9}{7}\right)^x}{\ln\left(\frac{9}{7}\right)} + K}$$

Compito 16/10/21

~~102~~ - ~~104~~ - ~~106~~ - ~~111~~ - ~~113~~ - ~~114~~ 1281
~~98~~ 1280

E102 P1281

$$\int e^{x+2}\,dx = \int e^x \cdot e^2\,dx = e^2 e^x + c = \boxed{e^{x+2} + c}$$

E104 P1281

$$\int (5-e^x)\,dx = 5\int dx - \int e^x\,dx = \boxed{5x - e^x + c}$$

E106 P1281

$$\int (2^x + 2e^x + 2\cdot 4^x)\,dx = \frac{2^x}{\ln(2)} + 2e^x + 2\cdot\frac{4^x}{\ln(4)} + c = \frac{2^x}{\ln(2)} + 2e^x + 2\cdot\frac{4^x}{\ln(2)^2}$$

$$= \frac{2^x}{\ln(2)} + 2e^x + \frac{\cancel{2}\cdot 4^x}{\cancel{2}\ln 2} + c = \boxed{\frac{2^x}{\ln(2)} + 2e^x + \frac{4^x}{\ln 2} + c}$$

E111 P1281

$$\int e^x(1-2xe^{-x})\,dx = \int e^x - 2xe^0 = \int e^x - 2x = e^x - \frac{\cancel{2}x^2}{\cancel{2}} + c = \boxed{e^x - x^2 + c}$$

E113 P1281

$$\int 8^x \cdot 2^{-3x+4}\,dx = \int 8^x \cdot 2^{-3x} \cdot 2^4\,dx = \int 2^{3x} \cdot 2^{-3x} \cdot 2^4\,dx = \int 2^4\,dx = \boxed{16x + c}$$

E114 P1281

$$\int 4^{x-1} \cdot 2^{-x+2}\,dx = \int (2^2)^{x-1} \cdot 2^{-x+2}\,dx = \int 2^{2x-2} \cdot 2^{-x+2}\,dx = \int 2^{2x} \cdot \cancel{2^{-2}} \cdot 2^{-x} \cdot \cancel{2^{2}}\,dx =$$

$$= \int 2^x\,dx = \boxed{\frac{2^x}{\ln(2)} + c}$$

E98 P1280

$C'(x) = \frac{x^2}{10'000} - \frac{x}{10} + 20$ per $x \leq 200$ $K = 0$

a) $C(x) = ?$

b) costo per produrre 100 ?

a) $$\int \left(\frac{x^2}{10'000} - \frac{x}{10} + 20 \right) dx = \frac{1}{10^4} \frac{x^3}{3} - \frac{1}{10} \cdot \frac{x^2}{2} + 20x =$$

$$\boxed{\frac{x^3}{3 \cdot 10^4} - \frac{x^2}{20} + 20x + K}$$

b) $$C(100) = \frac{(10^2)^3}{3 \cdot 10^4} - \frac{(10^2)^2}{2 \cdot 10^1} + 2 \cdot 10^1 \cdot (10^2)$$

$$= \frac{1}{3} \cdot \frac{10^{\cancel{6}\,2}}{\cancel{10^4}} - \frac{10^{\cancel{4}\,3}}{\cancel{10}} \cdot \frac{1}{2} + 2 \cdot 10^3$$

$$= \frac{1}{3} \cdot 10^2 - 10^3 \cdot \frac{1}{2} + 2 \cdot 10^3$$

$$= \frac{100}{3} - 500 + 2000$$

$$= \frac{100 - 1500 + 6000}{3}$$

$$= \frac{4600}{3}$$

$$= \boxed{1533.\overline{3} \text{ €}}$$

In classe

E119

$$\int \frac{\sin(x) - \sqrt{3}\cos(x)}{2}\, dx = \frac{1}{2}\int \sin(x)\, dx - \frac{\sqrt{3}}{2}\int \cos(x)\, dx =$$

$$-\frac{1}{2}\cos(x) - \frac{\sqrt{3}}{2}\sin(x) + K = \boxed{\frac{-\cos(x) - \sqrt{3}\sin(x)}{2} + K}$$

E122

$$\int \left(\frac{3}{\sin^2(x)} + \frac{\cos(x)}{2} \right) dx = \boxed{-\cot(x) + \frac{\sin(x)}{2} + K}$$

E125

$$\int \left(\frac{2}{\sin^2(x)} + \frac{1}{x} - \sin(x) \right) dx = -2\cot(x) + \ln|x| + \cos(x) + K$$

E127

$$\int \frac{-2\sin(2x)}{\cos(x)} = \int \frac{-2 \cdot 2\sin(x)\cancel{\cos(x)}}{\cancel{\cos(x)}}\, dx = \boxed{4\cos(x) + K}$$

E129

$$\int \tan^2(x)\, dx = \int \frac{\sin^2(x)}{\cos^2(x)}\, dx = \frac{1-\cos^2(x)}{\cos^2(x)}\, dx = \left(\frac{1}{\cos^2 x} - 1 \right) dx =$$

$$= \boxed{\tan(x) - x + K}$$

E130

$\cos^2 x - \sin^2 x$

$2\cos^2 x - 1$

$1 - 2\sin^2 x$

$$\int \frac{\cos(2x)}{4\cos^2(x)}\, dx = \int \left(\frac{2\cos^2 x}{4\cos^2 x} + \frac{-1}{4\cos^2 x} \right) dx = \boxed{\frac{1}{2}x - \frac{1}{4}\tan(x) + K}$$

E132

$$\int \frac{3}{\sqrt{1-x^2}}\,dx = \boxed{3\arcsin(x) + K}$$

E133

$$\int \frac{1}{4+4x^2}\,dx = \boxed{\frac{1}{4}\arctan(x) + K}$$

E134

$$\int \left(1 - \frac{6}{9-9x^2}\right) dx = \int \left(1 - \frac{2}{\sqrt{1-x^2}}\right) dx = \boxed{x - 2\arcsin(x) + K}$$

E136

$$\int \left(\frac{1}{x} + \frac{1}{1+x^2}\right) dx = \boxed{\ln|x| + \arctan(x) + K}$$

E140

$$\int \frac{-x^2}{1+x^2}\,dx = \int \frac{-x^2-1+1}{1+x^2}\,dx = \int \overbrace{\underbrace{\frac{-x^2-1}{1+x^2}}_{=\frac{-(x^2+1)}{(x^2+1)} = -1}}^{-1}\,dx + \int \frac{1}{1+x^2}\,dx = -x + \arctan(x) + K$$

Compito 19.10.21

~~120~~ – ~~124~~ – ~~128~~ – ~~135~~ – ~~137~~ – ~~138~~ – ~~141~~ 1282

E120 P1282

$$\int\left(\frac{\sin(x)}{3} - 5\cos(x)\right)dx = \boxed{-\frac{\cos(x)}{3} - 5\sin(x) + c}$$

E124

$$\int\left(\frac{1}{\cos^2(x)} - \frac{2}{\sin^2(x)} + 3\sin(x)\right)dx = \boxed{\tan(x) + 2\cot(x) - 3\cos(x) + c}$$

E128

$$\int \frac{5\sin(x) + 2\sin(2x)}{\sin(x)}\,dx = \int \frac{5\sin(x) + 4\sin(x)\cos(x)}{\sin(x)}\,dx = \int 5 + 4\cos(x)\,dx =$$

$$= \boxed{5x + 4\sin(x) + c}$$

E135

$$\int\left(\frac{12}{1+x^2} - 4x\right)dx = \boxed{12\arctan(x) - 2x^2 + c}$$

E137

$$\int\left(\frac{2}{\sqrt{1-x^2}} + x^{-1/2}\right) = 2\arcsin(x) + \frac{x^{1/2}}{\frac{1}{2}} + c = \boxed{2\arcsin(x) + 2\sqrt{x} + c}$$

E138

$$\int\left(2^x - \frac{14}{\sqrt{1-x^2}}\right)dx = \frac{2^x}{\ln(2)} - 14\left(-\arccos(x)\right) + c = \boxed{\frac{2^x}{\ln(2)} + 14\arccos(x) + c}$$

E141

$$\int \frac{1+2x^2}{1+x^2}\,dx = \int \frac{2x^2+2-2+1}{1+x^2}\,dx = \int \frac{2x^2+2}{1+x^2}\,dx + \int \frac{1-2}{1+x^2}\,dx =$$

$$\int \frac{2\cancel{(x^2+1)}}{\cancel{(x^2+1)}}\,dx + \int \frac{-1}{1+x^2}\,dx = \boxed{2x + \operatorname{arccot}(x) + c}$$

$$\parallel$$

$$\boxed{2x - \arctan(x) + c}$$

In classe

E144 P1283

$$\int \underbrace{5}_{f'(x)} \cdot \underbrace{(5x-2)^3}_{f(x)} \, dx = \boxed{\frac{(5x-2)^4}{4} + K}$$

E148 P1283

$$\int x^3 \left(4 - \frac{1}{2} x^4\right)^5 dx \rightarrow \int \frac{-2x^3}{-2} \left(4 - \frac{1}{2} x^4\right)^5 dx = \frac{\left(4 - \frac{1}{2}\right)^6}{6} \cdot -\frac{1}{2}$$

$f(x) = 4 - \frac{1}{2} x^4$

$f'(x) = -2x^3$ → aggiungo $\cdot \frac{-2}{-2}$

$$= \boxed{-\frac{\left(4 - \frac{1}{2}\right)^6}{12} + c}$$

E154 P1283

$$\int e^{2x} \sqrt{\underbrace{5 + e^{2x}}_{f(x)}} \, dx = \frac{1}{2} \cdot 2e^{2x} \left(5 + e^{2x}\right)^{1/2} = \left(5 + e^{2x}\right)^{3/2} : \frac{3}{2} \cdot \frac{1}{2} = \boxed{\frac{(5 + e^{2x})\sqrt{(5 + e^{2x})} + c}{3}}$$

$f'(x) = e^{2x} \cdot 2$

E163

$$\int \frac{\arcsin^4(x)}{\sqrt{1 - x^2}} \, dx = \boxed{\frac{\arcsin^5(x)}{5} + c}$$

$f(x) = \arcsin(x)$

$f'(x) = \frac{1}{\sqrt{1 - x^2}}$ OK

SE "n" $= -1$ devo usare "ln"

$$\int \underbrace{f(x)^{-1} \cdot f'(x)}\, dx = \ln|f(x)| + K$$

$$\frac{f'(x)}{f(x)}\, dx = \ln|f(x)| + K$$

E170 P1284

$$\int \frac{8x^3}{\underbrace{x^4+1}_{f(x)^{-1}}}\, dx = \int 2 \cdot \frac{4x^3}{x^4+1}\, dx = \boxed{2\ln|x^4+1| + K}$$

$f'(x) = 4x^3$

ES PROF 1

$$\int \tan(x)\, dx = \int \frac{\sin(x)}{\underbrace{\cos(x)}_{f(x)^{-1}}}\, dx = \int \frac{-1}{-1} \frac{\sin(x)}{\cos(x)}\, dx = \boxed{-\ln|\cos(x)| + K}$$

$f'(x) = \ominus \sin(x)$

↳ manca

ES PROF 2

$$\int \cot\text{an}(x)\, dx = \frac{\cos(x)}{\sin(x)}\, dx = \boxed{\ln|\sin(x)| + K}$$

$f(x) = \sin(x)$

$f'(x) = \cos(x)$ OK

E178

$$\int \frac{1}{x\ln(x)}\, dx = \int \frac{1}{x} \cdot \frac{1}{\ln(x)}\, dx = \boxed{\ln|\ln(x)| + K}$$

E181

$$\int \frac{7x + \arctan^3(x)}{1+x^2}\, dx = \int \frac{7x}{1+x^2} + \int \frac{\arctan^3(x)}{1+x^2} = \boxed{\frac{7}{2} \cdot \ln|1+x^2| + \frac{\arctan^4(x)}{4} + C}$$

$f(x) = 1+x^2$

$f'(x) = \boxed{2}x$

$f(x) = \arctan^3(x)$

$f'(x) = \dfrac{1}{1+x^2}$ OK

Compito 23.10.21

~~150~~ - ~~158~~ - ~~160~~ - ~~166~~ - ~~169~~ - ~~171~~ - ~~172~~ - ~~179~~ - ~~180~~
1283

E150 P1283

$$\int 15(6-5x)^{1/2}\,dx = -3\cdot\frac{15}{-3}(6-5x)^{1/2}\,dx = \frac{-2\cdot\cancel{3}(6-5x)^{3/2}}{\cancel{3}} = \boxed{-2(6-5x)^{3/2}+c}$$

$f(x) = 6-5x$

$f'(x) = -5$

E158

$$\int \frac{(3\ln(x))^2}{x}\,dx = 9\frac{\ln^3(x)}{3}+c = \boxed{3\ln^3(x)+c}$$

E160

$$\int \underbrace{\cos^3(x)}_{f(x)}\cdot\sin(x)\,dx = \boxed{\frac{-\cos^4(x)}{4}+c}$$

$f'(x) = \boxed{-}\sin(x)$

E166

$$\int \frac{\overset{f'(x)}{6x}}{\underset{f(x)}{3x^2+4}}\,dx = \boxed{\ln|3x^2+4|+c}$$

$f'(x) = 2\cdot 3x$ OK

E169

$$\int \frac{x^2}{\underset{f(x)}{x^3+2}}\,dx = \int\frac{1}{3}\cdot\frac{3x^2}{x^3+2}\,dx = \boxed{\frac{1}{3}\ln|x^3+2|+c}$$

$f'(x) = 3x^2$ $\cdot 3/3$

E171

$$\int \frac{x+1}{\underset{f(x)}{x^2+2x-3}}\,dx = \int\frac{1}{2}\,\frac{2(x+1)}{x^2+2x-3}\,dx = \boxed{\frac{1}{2}\ln|x^2+2x-3|+c}$$

$f'(x) = 2x+2 = \boxed{2}(x+1)$ $\cdot 2/2$

E172

$$\int \underbrace{\frac{3x^2+4x+1}{x^3+2x^2+x}}_{f(x)}\,dx = \boxed{\ln|x^3+2x^2+x| + c}$$

$f'(x) = 3x^2+4x+1$ OK

E179

$$\int \underbrace{\frac{2\cdot x^{-1/2}}{(1+\sqrt{x})}}_{f(x)}\,dx = \int \frac{1}{2\sqrt{x}} \cdot \frac{1}{1+\sqrt{x}} \cdot 2 \cdot 2 = \boxed{4\ln|1+\sqrt{x}| + c}$$

$f'(x)\ \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}\ \cdot\frac{2}{2}$

E180

$$\int \underbrace{\frac{\tan(x)}{1+\ln(\cos(x))}}_{f(x)}\,dx = \boxed{-\ln|1+\ln(\cos(x))| + c}$$

$f'(x) = \frac{1}{\cos(x)} \cdot (-\sin(x)) = \frac{\boxed{-}\sin(x)}{\cos(x)}\ \cdot\frac{-1}{-1}$

In classe - ancora funzioni composte

E 185

$$\int 3e^{\overset{f(x)}{\boxed{-3x}}}\,dx = -\int -3e^{-3x}\,dx = \boxed{-e^{-3x} + K}$$

$f'(x) = E13$ $\quad \cdot \frac{-1}{-1}$

$$\int e^{f(x)} \cdot f'(x)\,dx = e^{f(x)} + c$$

E 190

$$\int e^{(x^2+x)}(2x+1)\,dx = \boxed{e^{(x^2+x)} + K}$$

$(x^2+x) = t$

$dt = t' \cdot dx \qquad dt = (2x+1)\,dx$

$dx = \dfrac{dt}{(2x+1)}$

$$\int e^t \cancel{(2x+1)} \cdot \frac{dt}{\cancel{(2x+1)}} = \int e^t \cdot dt = \boxed{e^{(2x+1)} + c}$$

E 198

$$\int \frac{3^{\ln(x)}}{x}\,dx = \frac{3^{\ln(x)}}{\ln(3)} + K \qquad *$$

$D(\ln x) = \dfrac{1}{x}$

EX PROF

$$\int \sin(x^3) \cdot 3x^2\,dx = \boxed{-\cos(x^3) + K}$$

Per sostituzione:

$x^3 = t$

$dt = 3x^2 \cdot dx \rightarrow dx = \dfrac{dt}{3x^2}$

$$\int \sin(t) \cdot \cancel{3x^2} \cdot \frac{dt}{\cancel{3x^2}} = \boxed{-\cos(x^3) + K}$$

ES A CASO

$$\int \frac{\sin e^{-x}}{e^x}\,dx = \int -1 \cdot \frac{\sin e^{-x}}{e^x} = -(-\cos e^{-x}) + K = \boxed{\cos(e^{-x}) + K}$$

$f(x) = e^{-x}$

$f'(x) = \boxed{-}e^{x}$

Per sostituzione

$e^{-x} = t$

$-e^{-x}\,dx = dt$

$dx = \dfrac{dt}{-e^{-x}}$

$$\int \frac{\sin(t)}{\cancel{e^{x}}} \cdot \left(\frac{-dt}{\cancel{e^{-x}}}\right) = \int -(\sin(t))\,dx = \boxed{\cos(e^{-x}) + C}$$

E212

$$\int \frac{4x+1}{\sin^2(\underbrace{2x^2+x}_{f(x)})}\,dx = \boxed{-\cot\text{an}(2x^2+x) + c}$$

$f'(x) = 4x+1$

E218

$$\int \frac{4x}{\sqrt{1-4x^2}}\,dx = \boxed{\arcsin(2x^2) + K}$$

RICORDA

$$\int \frac{1}{\sqrt{1-4x^2}}\,dx \longrightarrow \int \frac{1}{\sqrt{1-f^2(x)}} \cdot f'(x)\,dx = \arcsin(f x) + c$$

$f^2(x) = 4x^2 \longrightarrow f(x) = 2x^2$

$\downarrow$

$f'(x) = \boxed{4x}$ OK

E226

$$\int \frac{\sin(x)}{4+\cos^2(x)}\,dx = \int \frac{\sin x}{4\left(1+\boxed{\frac{\cos^2(x)}{4}}\right)}\,dx = \frac{-\frac{1}{2}\sin(x)}{-\frac{1}{2}\cdot 4\left(1+\frac{\cos^2(x)}{4}\right)}\,dx =$$

$$\int \frac{1}{1+x^2}\,dx = \arctan$$

$f^2(x)$

$f(x) = \frac{\cos(x)}{2}$

$f'(x) = \boxed{\frac{-\sin(x)}{2}}$

$$\boxed{= -\frac{1}{2}\arctan\left(\frac{\cos x}{2}\right)}$$

In classe pre verifica

$C_{MAX} = 12\ mg/L$

$c'(t) = -4e^{-t/3}$

t = tempo in ore

- $c(t) = ?$
- quante "h" per dimezzare C

$$C(t) = \int -4e^{-t/3}\,dx = 3\cdot 4 \int -\frac{1}{3}\, e^{-t/3}\,dt = \boxed{12\,e^{-t/3} + K}$$

$f(x) = -\frac{t}{3}$

$f'(x) = -\frac{1}{3}$ manca

trovare "K"

$C(t) = 12\,e^{-t/3} + K$

$c(0) = 12 = 12\cdot e^{0} + K \longrightarrow 12 = 12 + K \longrightarrow K = 0$

↓

valore massimo a tempo = 0

$c(t) = 12\,e^{-t/3}$

$\frac{6}{12} = \frac{12}{12}\,e^{-\frac{t}{3}}$

$e^{-\frac{t}{3}} = \frac{1}{2}$

$-\frac{t}{3} = \ln\left(\frac{1}{2}\right)$

$t = -3\ln(0.5) \boxed{\simeq 2\ h}$

E320

$v_0 = 15\ m/s$

$h_0 = 1\ m$

$g = -10\ m/s$

$h(t) = \frac{1}{2} g t^2 + v_0 t + 1$ legge oraria

$h(t) = \frac{1}{2}(-10)t^2 + 15t + 1$

$h(t) = -5t^2 + 15t + 1$

COME RICAVARE LA LEGGE ORARIA

$h(t) = \int v(t)\, dt$

$v(t) = \int g(t)\, dt = \int -10\, dt = -10t + K$ è la velocità iniziale

$v(t) = -10t + K$

$v(0) = v_0 = 15 = K$

$V(t) = -10t + 15$

$h(t) = \int (-10t + 15)\, dt = -\frac{10t^2}{2} + 15t + K$

$\boxed{h(t) = -5t^2 + 15t + K}$

$h(0) = 1 = K \rightarrow \boxed{h(t) = -5t^2 + 15t + 1}$

dopo 1 secondo

$h(1) = -5 + 15 + 1 = \boxed{11 \text{ metri}}$

E321

$N'(t) = 250\, t^{3/2}$

$N(t) = \int 250\, t^{3/2}\, dt = \frac{2}{5} \cdot 250\, t^{5/2} + \overset{K}{\overset{\parallel}{0}}$

$\boxed{N(t) = 100\, t^{5/2} = 100\, t^2 \sqrt{t}}$

$N(12) = 10\,(12)^{5/2} \simeq \boxed{5'000 \text{ litri}}$

E318

$f(x) = 3x^2 - x$

$F(x) = ?$ con max di ordinata 2

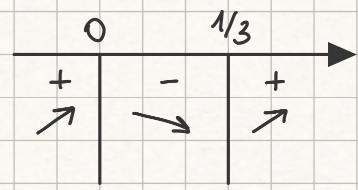
$F(x) = \int (3x^2 - x)\, dx = x^3 - \frac{x^2}{2} + K$

segno di $f(x)$ mi da il crescere di $F(x)$

$f(x) = 3x^2 - x > 0$

$x(3x-1) > 0$

$x < 0 \lor x > 1$



ABBIAMO IL MASSIMO IN $x = 0$

$2 = x^3 - \frac{x^2}{2} + K$

$2 = 0 - 0 + K$

$\boxed{K = 2}$

E 303

$$\int \frac{\cos x \cdot e^{\overset{f(x)}{\sqrt{\sin x}}}}{2\sqrt{\sin x}}\,dx = f'(x)\cdot e^{f(x)} = \boxed{e^{\sqrt{\sin x}} + K}$$

$$f(x) = \sqrt{\sin x} = (\sin x)^{1/2}$$

$$f'(x) = \frac{1}{2}(\sin x)^{-1/2} \cdot \cos(x) = \frac{\cos(x)}{2\sqrt{\sin x}}$$

Pre verifica

~~262~~ - ~~267~~ - ~~268~~ - ~~269~~ - ~~271~~ - ~~273~~ - ~~279~~ - ~~280~~ - ~~282~~ - ~~283~~

E262 P1288

$$\int 3x^2 \cdot \sin(4x^3)\,dx = \int \frac{1}{4}\cdot 4 \cdot 3x^2 \sin(4x^3)\,dx = \frac{1}{4}\left(-\cos(4x^3)\right) + C$$

$f'(x) = 12x^2$

$$= \boxed{-\frac{\cos(4x^3)}{4} + c}$$

E267

$$\int \frac{3\sin(x) - 2\cos(x)}{4}\,dx = \frac{3}{4}\int \sin(x)\,dx - \frac{1}{2}\int \cos(x)\,dx =$$

$$\boxed{-\frac{3}{4}\cos(x) - \frac{1}{2}\sin(x) + c}$$

E268

$$\int \left(\frac{2}{\cos^2 x} - \frac{1}{\sin^2(x)}\right) dx = 2\int \frac{1}{\cos^2(x)}\,dx - \int \frac{1}{\sin^2(x)} =$$

$$\tan(x) - (-\cot(x)) + c = \boxed{\tan(x) + \cot(x) + c}$$

E269

$$\int \frac{2\cos(x) + \sin(2x)}{\cos(x)}\,dx = \int \frac{2\cancel{\cos(x)} + 2\sin(x)\cancel{\cos(x)}}{\cancel{\cos(x)}}\,dx = \boxed{2x - 2\cos(x) + c}$$

E271

$$\int \left(\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{1-x^2}}\right) dx = \int x^{-1/2}\,dx - \int \frac{1}{\sqrt{1-x^2}}\,dx = \frac{x^{1/2}}{1/2} - \arcsin(x) + c$$

$$\boxed{= 2\sqrt{x} - \arcsin(x) + c}$$

E273

$$\int \frac{1}{x\sqrt{1-\ln^2 x}}\,dx = \int \frac{1}{x} \cdot \frac{1}{\sqrt{1-\ln^2(x)}}\,dx = \boxed{\arcsin(\ln(x)) + c}$$

$f^2(x) \longrightarrow f(x) = 1 - \ln(x)$

$f'(x) = \frac{1}{x}$ ok

E279

$$\int (x^2+1)\sin(x^3+3x)\,dx = \int \frac{1}{3}\cdot 3\,(x^2+1)\sin(x^3+3x)\,dx =$$

$f'(x) = 3x^2+3$

$$= \boxed{-\frac{1}{3}\cos(x^3+3x)+c}$$

E280

$$\int (\sin(x))^5\cos(x)\,dx = \boxed{\frac{\sin^6(x)}{6}+c}$$

$f'(x) = \cos(x)$ ok

E282

$$\int (2x-1)^8\,dx = \int \frac{1}{2}\cdot 2\,(2x-1)^8\,dx = \boxed{\frac{1}{2}\,\frac{(2x-1)^9}{9}+c}$$

$f'(x) = \boxed{2}$

E283

$$\int \frac{x}{(\sin(x^2))^2}\,dx = \int \frac{1}{2}\,2x\,\frac{1}{\sin^2(x^2)}\,dx = \boxed{-\frac{1}{2}\cot(x^2)+c}$$

$f'(x) = \boxed{2}x$

$f(x)$

~~285~~ - ~~286~~ - ~~287~~ - ~~288~~ - ~~290~~ - ~~293~~ - ~~295~~ - ~~298~~ - ~~299~~ - ~~300~~

E285 P1288

$$\int \frac{e^{\sqrt{x}-4}}{\sqrt{x}}\,dx = \int 2\cdot\frac{1}{-2\sqrt{x}}\,e^{\sqrt{x}-4}\,dx = \boxed{2e^{\sqrt{x}-4}+c}$$

$f(x) = (x)^{1/2}$

$f'(x) = -\frac{1}{2}(x)^{-1/2} = \boxed{\frac{-1}{2}}\,\frac{1}{\sqrt{x}}$

E286

$$\int \frac{\arcsin(x)}{4\sqrt{1-x^2}}\,dx = \int \frac{1}{4}\,\underbrace{\arcsin(x)}_{}^{f(x)}\,\frac{1}{\sqrt{1-x^2}}\,dx = \boxed{\frac{1}{4}\cdot\frac{\arcsin^2(x)}{2}+c}$$

$f'(x) = \frac{1}{\sqrt{1-x^2}}$ OK

E287

$$\int \frac{\cos(x)}{9+\sin^2(x)}\,dx = \int \frac{\cos(x)}{9\left(1+\underbrace{\frac{\sin^2(x)}{9}}_{f^2(x)}\right)} = \int \frac{3\cos(x)}{3\cdot 9\left(1+\frac{\sin^2(x)}{9}\right)}\,dx = \boxed{\frac{\arctan}{3}\left(\frac{\sin(x)}{3}\right)+c}$$

$$f(x) = \frac{\sin(x)}{3} \rightarrow f'(x) = \frac{\cos(x)}{\boxed{3}}$$

E288

$$\int \frac{e^{x^{-2}}}{x^3}\,dx = \int \frac{1}{-2}\cdot\frac{-2}{x^3}\,e^{x^{-2}}\,dx = \boxed{-\frac{1}{2}\,e^{x^{-2}}+c}$$

$$f'(x) = -2x^{-3} = \frac{\boxed{-2}}{x^3}$$

E290

$$\int \sin(x)\sec(x)\,dx = \int \sin(x)\cdot\frac{1}{\cos(x)}\,dx = \int \frac{-1}{-1}\,\frac{\sin(x)}{\cos(x)}\,dx = \boxed{-\ln|\cos(x)|+c}$$

$$f'(x) = \boxed{-}\sin(x)$$

RICORDA

$$\sec = \frac{1}{\cos(x)}$$

$$\csc = \frac{1}{\sin(x)}$$

E293

$$\int 6^{2\sin(x)+1}\cos(x)\,dx = \int 6^{2\sin(x)+1}\,2\cos(x)\cdot\frac{1}{2}\,dx = \frac{1}{2}\,\frac{6^{2\sin(x)+1}}{\ln(6)}+c$$

$$f'(x) = \boxed{2}\cos(x)$$

$$= \frac{\cancel{6}^{\,3}}{\cancel{2}}\cdot\frac{6^{2\sin(x)}}{\ln(6)}+c = \boxed{3\cdot\frac{6^{2\sin(x)}}{\ln(6)}+c}$$

E295

$$\int \frac{3x}{2}(4x^2-6)^{1/2}\,dx = \int \frac{3}{16}\,\frac{16}{3}\,\frac{3x}{2}(4x^2-6)^{1/2}\,dx = \frac{3}{16}\,\frac{(4x^2-6)^{3/2}}{\frac{3}{2}}+c =$$

$$f'(x) = \boxed{8}x \qquad \cdot \div \frac{16}{3}$$

$$\frac{3}{2}\cdot x = 8$$

$$x = \frac{16}{3}$$

$$= \frac{\cancel{3}}{\cancel{16}_{\,8}}\cdot\frac{\cancel{2}}{\cancel{3}}(4x^2-6)\sqrt{4x^2-6}+c = \boxed{\frac{\overset{2}{\cancel{4}}x^2-\overset{3}{\cancel{6}}}{\cancel{8}\,4}\sqrt{4x^2-6}+c}$$

E298

$$\int \frac{1}{(x-2)\ln(x-2)}\,dx = \int \frac{1}{t\ln(t)}\,dt = \int \frac{1}{\cancel{t}v}\cdot dv\cancel{t} = \int \frac{1}{v}\,dv =$$

SOSTITUZIONE

$(x-2)=t$
$dt = f'(t)\cdot dx$
$dt = dx$
$dx = dt$

SOSTITUZIONE

$\ln(t) = v$
$dv = f'(v)\cdot dt$
$dv = \frac{1}{t}\cdot dt$

$dt = dv\cdot t$

$$= \ln|v| + c = \ln|\ln(t)| + c = \boxed{\ln|\ln(x-2)| + c}$$

E299

$$\int \frac{x^4}{\sqrt{1-x^{10}}}\,dx = \int \frac{1}{5}\cdot\frac{5x^4}{\sqrt{1-x^{10}}} = \boxed{\frac{1}{5}\arcsin(x^5) + c}$$

$f(x) = x^5$
$f'(x) = \boxed{5}x^4$

E300

$$\int \frac{e^x\tan(e^x)}{(\cos(e^x))^2}\,dx =$$

SOSTITUZIONE :

$e^x = t$
$dt = f'(t)\cdot dx$
$dt = e^x\cdot dx$
$dx = \frac{dt}{e^x} \longrightarrow dx = \frac{dt}{t}$

$$\int \frac{\cancel{t}\tan(t)}{\cos^2(t)}\cdot\frac{dt}{\cancel{t}} = \int \frac{\tan(t)}{\cos^2(t)}\,dt = \int \underbrace{\tan(t)}_{f(x)}\cdot\frac{1}{\cos^2(t)}\,dt = \frac{\tan^2(t)}{2} + c$$

$f'(x) = \frac{1}{\cos^2(x)}$ OK

$$= \boxed{\frac{\tan^2(e^x)}{2} + c}$$

~~302~~ - ~~308~~ - ~~312~~ - 317

P1310 ~~54~~ - ~~55~~
P1281 ~~109~~ - ~~114~~

E 302

$f(x) \cdot f'(x)$

$$\int \underbrace{(\arctan(4x))}_{f(x)}^2 \cdot \frac{1}{1+16x^2}\,dx = \int \arctan^2(4x)\,\frac{4}{1+16x^2}\cdot\frac{1}{4}\,dx =$$

$$f'(x) = \frac{1}{1+16x^2}\cdot \boxed{4}$$

$$= \frac{\arctan^3(4x)}{3\cdot 4} + c = \boxed{\frac{1}{12}\arctan^3(4x) + c}$$

E 308

$\arctan = \frac{1}{1+x^2}$

$$\int \frac{1}{x(1+(\ln(x^3))^2)}\,dx = \int \frac{1}{x(1+\ln^2(t))}\cdot\frac{1}{3x^2}\cdot dt = \int \frac{1}{3x^3}\,\frac{1}{(1+\ln^2(t))}\,dt =$$

SOSTITUZIONE

$x^3 = t$

$dt = f'(t)\cdot dx$

$dt = 3x^2\,dx$

$dx = \frac{dt}{3x^2}$

$$= \frac{1}{3}\cdot\int\frac{1}{t}\cdot\frac{1}{1+(\ln(t))^2}\,dt =$$

SOSTITUZIONE

$\ln(t) = v$

$dv = f'(v)\cdot dt$

$dv = \frac{1}{t}\cdot dt$

$dt = dv\cdot t$

$$= \frac{1}{3}\int\frac{1}{\cancel{t}}\,\frac{1}{1+v^2}\,dv\cdot\cancel{t} =$$

$$= \frac{1}{3}\arctan(v) = \frac{1}{3}\arctan(\ln(t)) =$$

$$\boxed{= \frac{1}{3}\arctan(\ln(x^3)) + c}$$

E 312

$f(x) = 2e^x + x$

$F(x) = ?$ in $P(\overset{x}{0}; \overset{y}{-1})$

$F(x) = \int 2e^x + x \, dx = \boxed{2e^x + \frac{x^2}{2} + c}$

$-1 = 2e^0 + \cancel{\frac{0}{2}} + c$

$-1 = 2 + c$

$c = -3 \longrightarrow \boxed{F(x) = 2e^x + \frac{x^2}{2} - 3}$

E 317

$f(x) = y = 2\cos(2x)$

ascissa $[x] = \frac{\pi}{2}$ ammette come tangente la retta: $y = -2x + \pi + 2$

$F(x) = \int 2\cos(\underbrace{2x}_{f(x)}) \, dx = \boxed{\sin(2x) + c}$ tutte le primitive

$f'(x) = 2$ OK

$F(x) = \sin(2x) + c$

sappiamo che $x = \frac{\pi}{2}$

coef. angolare della retta tangente $= -2$

$m = -2 = F'\left(\frac{\pi}{2}\right) = 2\cos(\pi) = -2$

$y\left(\frac{\pi}{2}\right) = -2\frac{\pi}{2} + \pi + 2$

$y = -\pi + \pi + 2$

$y = 2$

E109 P1281

$$\int (2-e^x-5^x)\,dx = \boxed{2x-e^x-\frac{5^x}{\ln(5)}+c}$$

E114

$$\int 4^{x-1}\cdot 2^{-x+2}\,dx = \int (2^2)^{x-1}\cdot 2^{-x+2}\,dx = \int 2^{2x-2}\cdot 2^{-x+2}\,dx = \int 2^{2x-\cancel{2}-x+\cancel{2}}\,dx =$$

$$\int 2^x\,dx = \boxed{\frac{2^x}{\ln(2)}+c}$$

E54 P1310

$$D'(x) = -\frac{3000}{x^2}$$

$$D(x) = -3000\int x^{-2}\,dx = -3000\,\frac{1}{x}\cdot(-1)+C = \boxed{\frac{3000}{x}+c}$$

$x = 3$ € $D = 1500$ $c = ?$

$$1500 = \frac{3000}{3}+c$$

$$1500 = 1000 + c$$

$$c = 500 \longrightarrow \boxed{D(x) = \frac{3000}{x}+500}$$

$x = 4$ € $c = 500$ $D(x) = ?$

$$D(x) = \frac{3000}{4}+500 =$$

$= 750 + 500 = \boxed{1250}$ confezioni richieste

ES5

$t_0 \rightarrow T = 90° C$

$T_a = 20° C$

$$T'(t) = -14 e^{-\frac{t}{5}}$$

$$T(t) = \int -14 e^{-t/5} dx = -14 \int (-5) - \frac{1}{5} \cdot e^{-\frac{t}{5}} dx = \boxed{70 e^{-\frac{t}{5}} + 20}$$

$f'(x) = -\frac{1}{5}$

se lo voglio a $T = 40° C$ $\quad t = ?$

$$40 = 70 e^{-\frac{t}{5}} + 20$$

$$\frac{20}{70} = e^{-\frac{t}{5}}$$

$$-\frac{t}{5} = \ln\left(\frac{2}{7}\right)$$

$t = -5 \ln(2/7) \sim$ **6 minuti**

prevede che se passa tanto tempo il thè arriverà (tenderà) alla temp. ambiente di 20° C.

E 300

$$\int \frac{e^x \tan(e^x)}{(\cos(e^x))^2} \, dx =$$

$\tan(f(x))$
$f(x) = e^x$
$f'(x) = e^x$ OK

$[g(f(x))]^n$ ← con $n=1$

$g(f x) = \tan(x)$
$g'(f x) = \frac{1}{\cos^2 x}$

$$\frac{[\tan(e^x)]^{n+1}}{n+1} = \frac{[\tan(e^x)]^2}{2} + c$$

E 303

$$\int \frac{\cos(x) \cdot e^{\sqrt{\sin x}}}{2\sqrt{\sin x}} \, dx = \boxed{e^{\sqrt{\sin x}} + c}$$

$f(x) = (\sin x)^{1/2}$

$f'(x) \; \frac{1}{2} (\sin x)^{-1/2} \cdot \cos x = \frac{\cos(x)}{2\sqrt{\sin x}}$ OK

E 181

$[f(x)]^n \cdot f'(x)$

$$\int \frac{7x + \arctan^3(x)}{1+x^2} \, dx = \int \frac{7x}{1+x^2} \, dx + \int \frac{\arctan^3(x)}{1+x^2} \, dx$$

$f(x)$
$f'(x) = \boxed{2}$

$$= \frac{7}{2} \int \frac{2x}{1+x^2} \, dx + \int \frac{\arctan^3(x)}{1+x^2} \, dx =$$

$$= \boxed{\frac{7}{2} \ln|1+x^2| + \frac{\arctan^4(x)}{4} + c}$$

E141

$$\int \frac{1+2x^2}{1+x^2}\,dx = \int \frac{+1-1+2x^2+1}{1+x^2} = \int \frac{2+2x^2}{1+x^2} - \int \frac{1}{1+x^2} = \boxed{2x - \arctan(x) + c}$$

mi ricorda $\frac{1}{1+x^2}$ "arctan(x)"

E305

$$\int \frac{1}{x}\cos\left(\ln \frac{1}{x^2}\right) dx$$

$\cos(f(x))$

$f(x) = \ln \frac{1}{x^2} = \ln(x^{-2})$

$f'(x) = \frac{1}{x^{-2}} \cdot (-2)\,x^{-3} = \frac{-2x^2}{x^3} = \frac{\boxed{-2}}{x}$

$$\int \frac{-2}{-2x}\cos\left(\ln \frac{1}{x^2}\right) dx = \boxed{-\frac{1}{2}\sin\left(\ln(1/x^2)\right) + c}$$

Pre verifica 2

E237 **P1285** ~~237~~ - ~~245~~ - ~~252~~ - ~~254~~ - ~~255~~ - ~~256~~ - ~~259~~ - ~~260~~ - ~~261~~ - ~~264~~

$$\int\left(\frac{3}{2}\sqrt{x}+3x^2-3\right)dx = \int \frac{\cancel{3}}{\cancel{2}}\, x\sqrt{x}\,\frac{\cancel{2}}{\cancel{3}} + \frac{\cancel{3}}{\cancel{3}}x^3 - 3x + c = \boxed{x\sqrt{x}+x^3-3x+c}$$

E245

$$\int\left(-3\cdot\frac{1}{x}+x^1\cdot x^{1/2}\right)dx = -3\int\frac{1}{x}\,dx+\int x^{3/2}\,dx = \boxed{-3\ln|x| + x^2\sqrt{x}\,\frac{2}{5}+c}$$

E252

$$\int\left(\frac{2}{x}+(x^{-1})^{1/2}+x^{1/3}\right)dx = \boxed{2\ln|x|+2\sqrt{x}+\frac{3x\sqrt[3]{x}}{4}+c}$$

E254

$$\int\left(x-2+\frac{1}{x}\right)(x+1)\,dx = \int\left(x^2+x-2x-2+1+\frac{1}{x}\right)dx = \int\left(x^2-x-1+\frac{1}{x}\right)dx =$$

$$= \boxed{\frac{x^3}{3}-\frac{x^2}{2}-x+\ln|x|+c}$$

E255

$$\int\frac{\sqrt{x}+2x+1}{4x}\,dx = \int\left(\frac{1}{4\sqrt{x}}+\frac{1}{2}+\frac{1}{4x}\right)dx = \frac{1}{\cancel{4}\,2}\sqrt{x}\cdot\cancel{2}+\frac{1}{2}x+\frac{1}{4}\ln(x)+c =$$

$$= \boxed{\frac{\sqrt{x}}{2}+\frac{x}{2}+\frac{\ln(x)}{4}+c}$$

E256

$$\int\frac{3}{3x+1}\,dx = \boxed{\ln|3x+1|+c}$$

$f(x)$

$f'(x) = 3$ ok

E259

$$\int\frac{3x^2-2}{\sqrt{2x^3-4x}}\,dx = \int(3x^2-2)\cdot(2x^3-4x)^{-1/2}\,dx = \frac{1}{2}\int 2(3x^2-2)(2x^3-4x)^{-1/2} =$$

$f(x)$

$f'(x) = 6x^2-4 = \boxed{2}(3x^2-2)$

NON È $\frac{f'(x)}{f(x)}$ perche' c'e' la radice!

$$= \boxed{\frac{1}{2}\,2\sqrt{2x^3-4x}+c}$$

E260

$$\int \frac{4x+2}{x^2+x}\,dx = \int \frac{2(2x+1)}{x^2+x}\,dx = \boxed{2\ln|x^2+x| + c}$$

$f(x)$

$f'(x) = 2x+1$ ok

E261

$$\int e^{-4x}\,dx = \int -\frac{1}{4}\cdot 4\,e^{-4x}\,dx = \boxed{-\frac{1}{4}\cdot e^{-4x} + c}$$

$f(x) = -4x$

$f'(x) = \boxed{-4}$

E264

$$\int [\sin(x)]^2 \cos(x)\,dx = \boxed{\frac{\sin^3(x)}{3} + c}$$

$f(x) = \sin(x)$

$f'(x) = \cos(x)$ ok

~~265~~ – ~~272~~ – ~~276~~ – ~~277~~ – ~~278~~ – ~~281~~ – ~~284~~ – ~~289~~ – ~~292~~ – ~~294~~

E265 P1288

$$\int \frac{(\ln(x))^3}{x}\,dx = \boxed{\frac{\ln^4(x)}{4} + c}$$

E272

$$\int \frac{\cos(x)+\sin(x)}{\sin(x)-\cos(x)}\,dx = \boxed{\ln|\sin(x)-\cos(x)| + c}$$

$f'(x) = \cos(x)+\sin(x)$ ok

E276

$$\int \frac{\cos(x)}{1+(\sin(x))^2}\,dx = \int \cos(x)\cdot\frac{1}{1+\sin^2(x)}\,dx = \boxed{\arctan(\sin(x)) + c}$$

$$\frac{1}{1+f^2(x)} = \arctan(fx)$$

$f(x) = \sin(x)$

$f'(x) = \cos(x)$ ok

E277

$$\int \frac{1}{(1+x^2)\arctan(x)}\,dx = \int [\arctan(x)]^{-1} \cdot \underbrace{\frac{1}{1+x^2}}_{f'(x)}\,dx = \boxed{\ln|\arctan(x)| + c}$$

ok

E278

$\cos^2 + \sin^2 = 1 \longrightarrow \sin^2 = 1 - \cos^2$

$$\int (2\tan^2(x) - 1)\,dx = 2\int \frac{\sin^2(x)}{\cos^2(x)}\,dx - \int 1\,dx = 2\int \frac{1-\cos^2(x)}{\cos^2(x)}\,dx - \int dx$$

$$= 2\int\left(\frac{1}{\cos^2(x)} - \frac{\cos^2(x)}{\cos^2(x)}\right)dx - \int dx = 2\int\left(\frac{1}{\cos^2(x)} - 1\right)dx - \int dx =$$

$$= 2(\tan(x) - x) - x + c = \boxed{2\tan(x) - 3x + c}$$

E281

$$\int \frac{4^{1+2x}}{8^x}\,dx = \int 4^{1+2x} \cdot 8^{-x}\,dx = \int 2^{2+4x} \cdot 2^{-3x}\,dx = \int 2^{x+2}\,dx =$$

$$= 2^2 \int 2^x\,dx = \boxed{4 \cdot \frac{2^x}{\ln(2)} + c}$$

E284

$$\int \frac{x^3}{\sqrt{1-x^8}}\,dx = \int x^3 \frac{1}{\sqrt{1-x^8}} = \int \frac{1}{4} \cdot 4x^3 \frac{1}{\sqrt{1-x^8}}\,dx =$$

$\frac{1}{\sqrt{1-f^2(x)}} = \arcsin(f(x))$

$f^2(x) = x^8$

$f(x) = x^4$

$f'(x) = \boxed{4}x^3$

$$\boxed{= \frac{1}{4}\arcsin(x^4) + c}$$

E289

$$\int \frac{1}{25+4x^2}\,dx = \int \frac{1}{25\left(1+\frac{4}{25}x^2\right)}\,dx = \int \frac{\cancel{5}}{2} \cdot \frac{2}{5} \frac{1}{\cancel{25}\left(1+\frac{4}{25}x^2\right)}\,dx$$

$\frac{1}{1+f^2(x)} = \arctan(f(x))$

$f^2(x) = \frac{4}{25}x^2$

$f(x) = \frac{2}{5}x$

$f'(x) = \boxed{\frac{2}{5}}$

$$\boxed{= \frac{1}{10}\arctan\left(\frac{2}{5}x\right) + c}$$

E292

$$\int \frac{1}{\sqrt{16-9x^2}}\,dx = \int \frac{1}{\sqrt{16\left(1+\frac{9}{16}x^2\right)}}\,dx = \int \frac{1}{|4|\sqrt{1+\frac{9}{16}x^2}}\,dx =$$

$$\frac{1}{\sqrt{1-f^2(x)}} = \arcsin(f(x))$$

$$f^2(x) = \frac{9}{16}x^2$$

$$f(x) = \frac{3}{4}x \qquad f'(x) = \boxed{\frac{3}{4}}$$

$$\int \frac{1}{\cancel{4}} \cdot \frac{\cancel{4}}{3} \cdot \frac{3}{4} \cdot \frac{1}{\sqrt{1+\frac{9}{16}x^2}}\,dx = \boxed{\frac{1}{3}\arcsin\left(\frac{3}{4}x\right) + c}$$

E294

$$\int \frac{x^4+\ln(x)}{x}\,dx = \int x^3\,dx + \int \ln(x)\cdot\frac{1}{x}\,dx = \boxed{\frac{x^4}{4} + \frac{\ln^2(x)}{2} + c}$$

~~297~~ – ~~304~~ – ~~306~~ – ~~195~~ – ~~196~~ – ~~205~~ – ~~206~~ – ~~210~~ – ~~213~~ – ~~215~~ – ~~221~~ – ~~225~~

E297 P1288

$$\int\left(\frac{16^x}{2^{3x+2}} + 3x^2\right)dx = \int\left(2^{4x}\cdot 2^{-3x-2} + 3x^2\right)dx = \int(2^{x-2} + 3x^2)\,dx =$$

$$= \frac{1}{4}\,\frac{2^x}{\ln(2)} + x^3 + c =$$

$$= \frac{2^x}{4\ln(2)} + x^3 + c =$$

$$= \frac{2^x}{\ln(2)^4} + x^3 + c =$$

$$= \boxed{\frac{2^x}{\ln(16)} + x^3 + c}$$

E304

$$\int \frac{2x-1}{\sqrt{1-4x^2}}\,dx = \int \frac{2x}{\sqrt{1-4x^2}}\,dx + \int \frac{-1}{\sqrt{1-4x^2}}\,dx =$$

$$\int 2x\cdot(1-4x^2)^{-1/2}\,dx + \int \frac{-1}{\sqrt{1-4x^2}}\,dx =$$

$f(x)$

$f'(x) = -8x$

$$\int -\frac{1}{4}\cdot 4\,2x(1-4x^2)^{-1/2}\,dx - \int \frac{1}{\sqrt{1-4x^2}}\,dx =$$

$f^2(x) = 4x^2$

$f(x) = 2x$

$f'(x) = \boxed{2}$

$$\int -\frac{1}{4}\cdot 4\,2x(1-4x^2)^{-1/2}\,dx - \int \frac{1}{2}\,2\,\frac{1}{\sqrt{1-4x^2}}\,dx =$$

$$-\frac{1}{\cancel{4}}\sqrt{1-4x^2}\cdot\cancel{2} - \frac{1}{2}\arcsin(2x) + c = \boxed{-\frac{1}{2}\left(\sqrt{1-4x^2} + \arcsin(2x)\right) + c}$$

E306

$$\int \left(\frac{x^2}{x-1}\right)^3 \cdot \frac{x^2-2x}{(x-1)^2}\,dx = \boxed{\frac{1}{4}\left(\frac{x^2}{x-1}\right)^4 + c}$$

$f(x)$

$f'(x) = \frac{2x(x-1)-x^2(1)}{(x-1)^2} = \frac{2x^2-2x-x^2}{(x-1)^2} = \frac{x^2-2x}{(x-1)^2}$ OK

E195

$e^{f(x)}$

$$\int e^{x\sin x}(\sin(x) + x\cos(x))\,dx = \boxed{e^{x\sin x} + c}$$

$f'(x) = \sin(x) + x\cos(x)$ OK

E196

$f(x)$

$$\int e^{\frac{1}{x}}\cdot\frac{1}{x^2}\,dx = \int -1\cdot -1\,\frac{e^{\frac{1}{x}}}{x^2}\,dx = \boxed{-e^{1/x} + c}$$

$f'(x) = x^{-1} = -x^{-2} = \boxed{-}\frac{1}{x^2}$

E 205

$$\int \frac{\sin(\overset{f(x)}{\sqrt{x}})}{\sqrt{x}}\,dx = \int 2 \cdot \frac{1}{2\sqrt{x}} \cdot \sin(\sqrt{x})\,dx = \boxed{-2\cos(\sqrt{x}) + c}$$

$$f'(x) = (x)^{1/2} = \frac{1}{2}x^{-1/2} = \boxed{\frac{1}{2}}\frac{1}{\sqrt{x}}$$

E 206

$$\int (x+2)\cos(\overset{f(x)}{x^2+4x})\,dx = \int \frac{1}{2}\cdot 2(x+2)\cos(x^2+4x)\,dx = \boxed{\frac{\sin(x^2+4x)}{2} + c}$$

$$f'(x) = 2x+4 = \boxed{2}(x+2)$$

E 210

$$\int \frac{e^x}{\cos^2(e^x)}\,dx = \boxed{\tan(e^x) + c}$$

$$\frac{f'(x)}{\cos^2(f(x))} = \tan(f(x))$$

$f(x) = e^x$

$f'(x) = e^x$ OK

E 213

$$\int \frac{1}{x\cos^2(\ln(x))}\,dx = \boxed{\tan(\ln(x)) + c}$$

$$\frac{f'(x)}{\cos^2(f(x))} = \tan(f(x))$$

$f(x) = \ln(x)$

$f'(x) = \frac{1}{x}$ OK

E215

$$\int \frac{2}{1+4x^2}\,dx = \int 2\cdot\frac{1}{1+4x^2}\,dx = \boxed{\arctan(2x)+c}$$

$$\frac{f'(x)}{1+f^2(x)} = \arctan(f(x))$$

$f'(x) = 2$ OK

E221

$$\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx = \boxed{\arcsin(e^x)+c}$$

E225

$$\int \frac{2}{9+4x^2}\,dx = \int 2\cdot\frac{1}{9\left(1+\frac{4}{9}x^2\right)}\,dx = \int \frac{\cancel{3}}{\cancel{9}}\cdot\frac{2}{3}\,\frac{1}{\left(1+\frac{4}{9}x^2\right)}\,dx =$$

$f^2(x) = \frac{4}{9}x^2$

$f(x) = \frac{2}{3}x$

$f'(x) = \frac{2}{3} = 2\cdot\boxed{\frac{1}{3}}$

$$= \boxed{\frac{1}{3}\arctan\left(\frac{2}{3}x\right)+c}$$

Prima Verifica

In classe: sostituzione

E 335

$$\int \frac{3}{\sqrt{x+2}}\,dx = \int \frac{3}{\cancel{\sqrt{x+2}}} \cdot 2\cancel{\sqrt{x+2}}\,dt = \boxed{6\sqrt{x+2} + K}$$

SOSTITUZIONE

$\sqrt{x+2} = t$

$dt = \frac{1}{2\sqrt{x+2}}\,dx$

$dx = 2\sqrt{x+2}\,dt$

E 339

$$\int \frac{1}{x-\sqrt{x}}\,dx = \int \frac{1}{t^2-t} \cdot 2t\,dt = \int \frac{1}{\cancel{t}(t-1)} \cdot 2\cancel{t}\,dt = 2\ln|t-1| + K =$$

$$= \boxed{2\ln|\sqrt{x}-1| + K}$$

SOSTITUZIONE

$\sqrt{x} = t \rightarrow x = t^2$

$dt = \frac{1}{2\sqrt{x}}\,dx$

$dx = 2\sqrt{x}\,dt$

E 340

$$\int \frac{1}{e^x + e^{-x}}\,dx = \int \frac{1}{t + \frac{1}{t}} \cdot \frac{dt}{t} = \int \frac{1}{\frac{t^2+1}{\cancel{t}}} \cdot \frac{dt}{\cancel{t}} = \int \frac{1}{t^2+1}\,dt =$$

SOSTITUZIONE

$e^x = t \rightarrow e^{-x} = 1/t$

$$= \arctan(t) + K = \boxed{\arctan(e^x) + K}$$

$dt = e^x\,dx$

$dx = \frac{dt}{e^x} = \frac{dt}{t}$

E344

$$\int \frac{x+3}{\sqrt{x+2}}\,dx = \int \frac{t^2-2+3}{\cancel{t}} \cdot 2\cancel{t}\,dt = \int (2t^2+2)\,dx = \frac{2t^3}{3} + 2t + K$$

$$= \frac{2\left[(x+2)^{1/2}\right]^3}{3} + 2\sqrt{x+2} + K =$$

S: $\sqrt{x+2} = t$

$dt = \dfrac{1}{2\sqrt{x+2}}\,dx$

$$= \frac{2(x+2)\sqrt{x+2}}{3} + 2\sqrt{x+2} + K$$

$dx = 2\sqrt{x+2}\,dt$

$x+2 = t^2 \longrightarrow x = t^2-2$

$$= 2\sqrt{x+2}\left(\frac{x}{3} + \frac{2}{3} + 1\right) + K$$

$$= \boxed{2\sqrt{x+2}\left(\frac{x+5}{3}\right) + K}$$

COMPITO 6.11.21

~~358~~ - ~~359~~ - ~~360~~ - ~~361~~ - ~~362~~ - ~~363~~ - ~~364~~ P1292

E358 P1292

$$\int \frac{\sin(x)}{3+2\cos(x)}\,dx = \int (3+2\cos(x))^{-1}(\sin(x))\,dx = \int \frac{1}{2}\cdot 2\sin(x)(3+2\cos(x))^{-1}\,dx =$$

$f'(x) = 2\sin(x)$

$$= \boxed{\frac{1}{2}\ln|3+2\cos(x)| + c}$$

E359

$$\int \frac{2\arctan(x)+1}{x^2+1}\,dx = \int \frac{2t+1}{\cancel{(x^2+1)}}\cancel{(1+x^2)}\,dt = \frac{\cancel{2}t^2}{\cancel{2}} + t + c =$$

SOSTITUZIONE $\arctan(x) = t$

$dt = \frac{1}{1+x^2}\,dx$

$dx = (1+x^2)\,dt$

$$= \boxed{\arctan^2(x) + \arctan(x) + c}$$

E360

$$\int \tan^3(x)\,dx = \int \frac{\sin^2(x)\sin(x)}{\cos^3(x)}\,dx = \int \frac{(1-\cos^2(x))\sin(x)}{\cos^3(x)}\,dx =$$

SOSTITUZIONE: $\cos(x) = t$

$dt = -\sin(x)\,dx$

$dx = \frac{-dt}{\sin(x)}$

$$= \int \frac{1-t^2}{t^3}\cdot\cancel{\sin(x)}\cdot\frac{dt}{\cancel{-\sin(x)}} =$$

$$= -1\int\left(\frac{1}{t^3} - \frac{1}{t}\right)dt =$$

$$= -1\left(\frac{t^{-2}}{-2} - \ln|t|\right) + c =$$

$$= \boxed{\frac{1}{2\cos(x)} + \ln|\cos(x)| + c}$$

E 361

$$\int \frac{\sin(x)}{4+\cos^2(x)}\,dx = \int \frac{\sin(x)}{4\left(1+\frac{\cos^2(x)}{4}\right)}\,dx = \frac{1}{4}\int \frac{\cancel{\sin}(x)}{\left(1+\frac{t^2}{4}\right)} \cdot \frac{dt}{-\cancel{\sin}(x)} =$$

$f(x) = \frac{t}{2}$ $\quad f'(x) = \boxed{\frac{1}{2}}$

SOSTITUZ: $\cos(x) = t$

$dt = -\sin(x)\,dx$

$dx = \frac{-dt}{\sin(x)}$

$$= -\frac{1}{4}\cdot 2\int \frac{2}{1+\frac{t^2}{4}}\,dt = -\frac{1}{2}\arctan\left(\frac{t}{2}\right)+c$$

$$= \boxed{-\frac{1}{2}\arctan\left(\frac{\cos(x)}{2}\right) + c}$$

E 362

$$\int \frac{2[e^x]^2}{1+e^x}\,dx = \int \frac{2e^{\cancel{2}x}}{t}\,\frac{dx}{\cancel{e^x}} = 2\int \frac{t-1}{t}\,dt = 2\left[\int dt + \int -\frac{1}{t}\,dt\right] =$$

SOSTIT: $e^x + 1 = t$

$dt = e^x\,dx$

$dx = \frac{dt}{e^x}$

$e^x = t-1$

$$= 2\left(t - \ln|t|\right) + c$$

$$= \boxed{2 + 2e^x - 2\ln|e^x+1| + c}$$

E 363

$$\int \frac{1}{\tan^3(x)}\,dx = \int \cot^3(x)\,dx = \int \frac{\cos^2(x)\cos(x)}{\sin^3(x)}\,dx = \int \frac{(1-\sin^2(x))\cos(x)}{\sin^3(x)} =$$

SOST: $\sin(x) = t$

$dt = \cos(x)\,dx$

$dx = \frac{dt}{\cos(x)}$

$$= \int \frac{(1-t^2)}{t^3}\,\cancel{\cos}(x)\cdot\frac{dt}{\cancel{\cos}(x)} =$$

$$= \int \frac{1}{t^3}\,dt + \int -\frac{1}{t}\,dt = \frac{1}{-2t^2} - \ln|t| + c =$$

$$= \boxed{\frac{1}{-2\sin(x)} - \ln|\sin(x)| + c}$$

E 364

$$\int \frac{\tan^3(x)+\tan(x)}{\tan(x)+2}\,dx = \int \frac{\tan^3(x)+\tan(x)}{t}\cdot\cos^2(x)\,dt =$$

SOST: $\tan(x)+2 = t$

$dt = \frac{1}{\cos^2(x)}\,dx$

$dx = dt\,\cos^2(x)$

$\tan(x) = t-2$

$$= \int \frac{\frac{\sin^3(x)}{\cos(x)} + \sin(x)\cdot\cos(x)}{t}\,dt =$$

$$= \int \frac{\frac{\sin^3(t)+\sin(x)\cos^2(x)}{\cos(x)}}{t}\,dt =$$

$$= \int \frac{\sin(x)}{\cos(x)}\left(\sin^2(x)+\cos^2(x)\right)\,dt =$$

$$= \int \tan(x)\left[1-\cancel{\cos^2(x)}+\cancel{\cos^2(x)}\right]dt =$$

$$= \int \frac{\tan(x)}{t}\,dt = \int \frac{\tan(x)+2-2}{t}\,dt =$$

$$= \int \frac{t-2}{t}\,dt = \int dt + \int -\frac{2}{t}\,dt =$$

$$= t - 2\ln|t| + c =$$

$$= \tan(x)+2-2\ln|\tan(x)+2| + c$$

$$= \boxed{2+\tan(x)-\ln(\tan(x)+2)^2 + c}$$

RICORDA:

$$D[\tan(x)] = \frac{\sin^2(x)+\cos^2(x)}{\cos^2(x)}$$

$$D[\tan(x)] = \tan^2(x)+1$$

veniva più comodo così

IN CLASSE: PARAMETRICHE

E370

$$\int \left(\cos(x) + \frac{4}{3\sin(x)} \right) dx$$

$$t = \tan\left(\frac{x}{2}\right) \qquad \sin(x) = \frac{2t}{1+t^2} \qquad \cos(x) = \frac{1-t^2}{1+t^2}$$

$$\arctan(t) = \frac{x}{2}$$

$$\frac{1}{1+t^2}\, dt = \frac{dx}{2}$$

$$dx = \frac{2}{1+t^2}\, dt$$

$$\int \left(\frac{1-t^2}{1+t^2} + \frac{4(1+t^2)}{6t} \right) \frac{2}{1+t^2}\, dt =$$

$$\int \frac{6t - 6t^3 + 4(1+t^2)^2}{6t(1+t^2)} \cdot \frac{2}{(1+t^2)}\, dt =$$

$$\int \frac{6t - 6t^3 + 4 + 8t^2 + 4^4}{6t(1+t^2)} \cdot \frac{2}{(1+t^2)}\, dt =$$

$$\int \frac{(8t^4 - 6t^3 + 8t^2 + 6t + 4) \cdot 2}{6t(1+t^2)^2}\, dt$$

E367

$$\int \frac{1}{\sin(x)-1}\,dx = \int \frac{1+t^2}{2t-1-t^2}\cdot\frac{2}{1+t^2}\,dt$$

$$\sin(x) = \frac{2t}{1+t^2}$$

$$dx = \frac{2}{1+t^2}\,dt$$

COMPITO: 09/11/21

~~365~~ - ~~366~~ - ~~367~~ - ~~368~~ - ~~369~~ - ~~370~~ - ~~371~~ - ~~372~~ - ~~373~~

ex: guida

P. 1293

E366

$$\int \frac{4}{1+\cos(x)}\,dx = \int \frac{4}{1+\frac{1-t^2}{1+t^2}}\cdot\frac{2}{1+t^2}\,dt = \int \frac{4\cancel{(1+t^2)}}{1-\cancel{t^2}+1+\cancel{t^2}}\cdot\frac{2}{\cancel{1+t^2}}\,dt$$

PARAMETRICHE: $\cos(x) = \frac{1-t^2}{1+t^2}$ con $t = x/2$

$$= \int \frac{4}{\cancel{2}}\cdot\cancel{2}\,dt =$$

$\frac{x}{2} = \arctan(t)$

$x = 2\arctan(t)$

$$= 4t + c =$$

$dx = \frac{2}{1+t^2}\,dt$

$$= \boxed{4\tan\left(\frac{x}{2}\right)+c}$$

E367

$$\int \frac{1}{\sin(x)-1}\,dx = \int \frac{1}{\frac{2t}{1+t^2}-1}\cdot\frac{2}{1+t^2}\,dt = \int \frac{\cancel{1+t^2}}{2t-1-t^2}\cdot\frac{2}{\cancel{1+t^2}}\,dt$$

$dx = \frac{2}{1+t^2}\,dt$ $\quad t = \tan(x/2)$

$$= \int \frac{1}{-t^2+2t-1}\cdot 2\,dt =$$

$\sin(x) = \frac{2t}{1+t^2}$

$$= \int \frac{2}{-(t-1)^2}\,dt = \frac{-2}{(t-1)^1}\cdot(-1)+c$$

$$= \boxed{\frac{2}{\left(\tan\left(\frac{x}{2}\right)-1\right)^3}+c}$$

E368

$$\int \frac{2}{\sin(x)}\,dx = \int \frac{2}{\frac{2t}{1+t^2}}\cdot\frac{2}{1+t^2}\,dt = \int \frac{2\cancel{(1+t^2)}}{\cancel{2}t}\cdot\frac{\cancel{2}}{\cancel{1+t^2}}\,dt =$$

$$= \boxed{2\ln\left|\tan(x/2)\right|+c}$$

PARAM: $\tan\left(\frac{x}{2}\right) = t$

$dx = \frac{2}{1+t^2}\,dt$

$\sin(x) = \frac{2t}{1+t^2}$

E369

$$\int \frac{2\sin^2(x)-1}{\sin(x)}\,dx = 2\int \sin(x)\,dx + \int \frac{-1}{\sin(x)}\,dx =$$

$$= 2\int -\cos(x)\,dx + \int \frac{-\cancel{(1+t^2)}}{\cancel{2}t}\cdot\frac{\cancel{2}}{\cancel{(1+t^2)}}\,dt =$$

$$= 2\int -\cos(x)\,dx + \int -\frac{1}{t}\,dt =$$

$$= -2\cos(x) - \ln|t| + c =$$

$$= \boxed{-2\cos(x) - \ln\left|\tan(x/2)\right| + c}$$

E370

$$\int\left(\cos(x) + \frac{4}{3\sin(x)}\right)dx = \int \cos(x)\,dx + \int \frac{4}{3}\cdot\frac{\cancel{1+t^2}}{\cancel{2}t}\cdot\frac{\cancel{2}}{\cancel{1+t^2}}\,dt =$$

$$= \int \cos(x)\,dx + \int \frac{4}{3t}\,dt =$$

$$= \boxed{\sin(x) + \frac{4}{3}\ln\left|\tan(x/2)\right| + c}$$

E371

$$\int \frac{3}{4(1+\sin(x))}\,dx = \int \frac{3}{4}\,\frac{1}{\left(1+\frac{2t}{1+t^2}\right)}\cdot\frac{2}{1+t^2}\,dt =$$

$$= \int \frac{3}{4}\,\frac{1}{\left(\frac{t^2+2t+1}{1+t^2}\right)}\,\frac{2}{1+t^2}\,dt =$$

$$= \int \frac{3\cancel{(1+t^2)}}{{}_2\cancel{4}(t+1)^2}\cdot\frac{\cancel{2}}{\cancel{(1+t^2)}}\,dt = \int \frac{3}{2}\cdot\frac{1}{(t+1)^2}\,dt =$$

$$= -\frac{3}{2}\cdot\frac{1}{(t+1)} + c = \boxed{-\frac{3}{2}\cdot\frac{1}{\left(\tan\left(\frac{x}{2}\right)+1\right)} + c}$$

E372

$$\int\left(\frac{1}{\sin(x)}-\overset{\overset{-2\cot(x)}{\Uparrow}}{\frac{2}{\sin^2(x)}}\right)dx = \int \frac{\cancel{1+t^2}}{\cancel{2}t}\cdot\frac{\cancel{2}}{\cancel{1+t^2}}\,dt + \int -2\cdot\frac{1}{\sin^2(x)}\,dx =$$

$$= \ln\left|\tan\left(\frac{x}{2}\right)\right| - (-2\cot(x)) + c =$$

$$= \boxed{\ln\left|\tan\left(x/2\right)\right| + 2\cot(x) + c}$$

E373

$$\int \frac{\sin(x)+3}{2\sin(x)}\,dx = \int \frac{1}{2}\,dx + \int \frac{3}{2}\cdot\frac{1}{\sin(x)}\,dx = \int \frac{1}{2}\,dx + \int \frac{3}{2}\,\frac{\cancel{1+t^2}}{\cancel{2}t}\cdot\frac{\cancel{2}}{\cancel{1+t^2}}\,dt =$$

$$= \boxed{\frac{1}{2}x + \frac{3}{2}\ln\left|\tan\left(x/2\right)\right| + c}$$

IN CLASSE: SOSTITUZIONE RAZIONALE

E375

$$\int \sqrt{9-x^2}\, dx =$$

$a^2 = 9$; $a = 3$ $\longrightarrow$ CE: $-3 \leq x < 3$

cambio variabile : $x = 3\sin(t)$ $\quad dx = 3\cos(t)\, dt$ $\qquad t = \arcsin\left(\frac{x}{3}\right)$

$$\int \sqrt{9-9\sin^2(t)} \cdot 3\cos(t)\cdot dt =$$

$$= 9\int \cos^2(t)\, dt$$

cambio variabile : $t = \frac{\alpha}{2}$ $\quad dt = \frac{1}{2}\, d\alpha$ $\quad \alpha = 2t$

$$= 9\int \cos^2\left(\frac{\alpha}{2}\right)\cdot \frac{1}{2}\, d\alpha$$

formula di bisezione : $\cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1+\cos(\alpha)}{2}}$

$$= 9\int \frac{1+\cos^2(\alpha)}{2}\cdot\frac{1}{2}\, d\alpha =$$

$$= \frac{9}{4}\int (1+\cos(\alpha))\, d\alpha = \frac{9}{4}(\alpha + \sin(\alpha)) + K$$

siccome $\alpha = 2t$

$$= \frac{9}{2}t + \frac{9}{4}\sin(2t) + K = \frac{9}{2}t + \frac{9}{2\cancel{4}}\cdot\cancel{2}\cos(t)\sin(t) + K =$$

$$= \frac{9}{2}t + \frac{9}{2}\cos(t)\sin(t) + K$$

siccome $t = \arcsin\left(\frac{x}{a}\right)$

$$= \frac{9}{2}\left(\arcsin\left(\frac{x}{3}\right) + \cos\left(\arcsin\left(\frac{x}{3}\right)\right)\overbrace{\sin\left(\arcsin\left(\frac{x}{3}\right)\right)}^{x/3\ =}\right) + K =$$

$$= \frac{9}{2} \arcsin\left(\frac{x}{a}\right) + \frac{\overset{3}{\cancel{9}}}{2} \cdot \frac{x}{\cancel{3}} \cdot \sqrt{1 - \sin^2\left(\arcsin\left(\frac{x}{3}\right)\right)} + K =$$

$$= \frac{9}{2} \arcsin\left(\frac{x}{a}\right) + \frac{3x}{2} \cdot \sqrt{1 - \frac{x^2}{a^2}} + K =$$

$$= \frac{9}{2} \arcsin\left(\frac{x}{a}\right) + \frac{\cancel{3}x}{2} \cdot \frac{1}{\cancel{3}} \sqrt{9 - x^2} + K =$$

$$= \frac{9}{2} \arcsin\left(\frac{x}{3}\right) + \frac{x}{2} \sqrt{9 - x^2} + K$$

E378 P1294

$$\int \sqrt{1-9x^2}\,dx = \int \sqrt{9\left(\frac{1}{9}-x^2\right)}\,dx = 3\int \sqrt{\underbrace{\frac{1}{9}-x^2}_{a^2-x^2}}\,dx$$

$a^2 - x^2 \longrightarrow a^2 = \frac{1}{9}\ ;\ a = \frac{1}{3}$

cambio variabile: $x = a \cdot \sin(t)$
$dx = a \cdot \cos(t) \cdot dt$
$t = \arcsin\left(\frac{x}{a}\right)$

$$= \cancel{3}\int \sqrt{\frac{1}{9} - \frac{1}{9}\sin^2(t)} \cdot \frac{1}{\cancel{3}}\cos(t)\,dt =$$

$$= \int \sqrt{\frac{\cos^2(t)}{9}} \cdot \cos(t)\,dt =$$

$$= \int \frac{\cos^2(t)}{3}\,dt$$

cambio variabile: $t = \frac{\alpha}{2}$ $\quad dt = \frac{1}{2}\,d\alpha$ $\quad \alpha = 2t$

$$= \frac{1}{3}\int \cos^2\left(\frac{\alpha}{2}\right) \cdot \frac{1}{2}\,d\alpha =$$

formula di bisezione: $\cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1+\cos(\alpha)}{2}}$

$$= \frac{1}{3}\int \frac{1+\cos(\alpha)}{2} \cdot \frac{1}{2}\,d\alpha =$$

$$= \frac{1}{12}\int (1+\cos(\alpha))\,d\alpha = \frac{1}{12}(\alpha + \sin(\alpha)) + K =$$

siccome $\alpha = 2t$

$$= \frac{1}{12}(2t + \sin(2t)) + K = \frac{1}{\cancel{12}\ 6}(\cancel{2}t + \cancel{2}\cos(t)\sin(t)) + K =$$

siccome $t = \arcsin\left(\frac{x}{a}\right)$

$$= \frac{1}{6}\left(\arcsin\left(\frac{x}{a}\right) + \cos\left(\arcsin\left(\frac{x}{a}\right)\right)\overbrace{\sin\left(\arcsin\left(\frac{x}{a}\right)\right)}^{= x/a}\right) + K =$$

$$= \frac{1}{6} \arcsin(3x) + \frac{1}{\cancel{6}_{2}} \cdot \cancel{3}x \cdot \sqrt{1-\sin^2(\arcsin(3x))} + K =$$

$$= \frac{1}{6} \arcsin(3x) + \frac{1}{2} x \cdot \sqrt{1-9x^2} + K$$

$$= \boxed{\frac{1}{6} \arcsin(3x) + \frac{x\sqrt{1-9x^2}}{2} + K}$$

IN CLASSE: INTEGRAZIONE PER PARTI

E388 P 1294

$$\int \underbrace{3x}_{f} \cdot \underbrace{\cos(x)}_{g'}\, dx = 3x \cdot \sin(x) - \int 3\sin(x)\, dx = 3x\sin(x) + 3\cos(x) + c$$

$f' = 3 \quad g = \sin(x)$

$$= \boxed{3(x\sin(x) + \cos(x)) + c}$$

E390

$$\int \frac{1}{x^2} \cdot \ln(x)\, dx = \underbrace{\ln(x)}_{f} \cdot \underbrace{x^{-2}}_{g'}\, dx = -\frac{\ln(x)}{x} - \int -\frac{1}{x} \cdot \frac{1}{x}\, dx =$$

$f' = 1/x \quad g = -x^{-1}$

$$= \frac{-\ln(x)}{x} - \frac{1}{x} + c =$$

$$= \boxed{-\frac{1}{x}(\ln(x) + 1) + c}$$

E392

$$\int \underbrace{4x}_{f}\, \underbrace{e^{2x}}_{g'}\, dx = \int 4x \cdot \frac{1}{2} \cdot 2e^{2x}\, dx = 4x \cdot \frac{e^{2x}}{2} - \int 4\frac{e^{2x}}{2}\, dx =$$

$f' = 4 \quad g = \frac{e^{2x}}{\boxed{2}}$

$$= 4x \cdot \frac{e^{2x}}{2} - e^{2x} + c$$

$$= \boxed{e^{2x}(2x - 1) + c}$$

COMPITO 13/12/21

~~387~~ - ~~389~~ - ~~393~~ P1294

E387 P1294

$$\int \underbrace{2x}_{g'}\,\underbrace{\ln(x)}_{f}\,dx = f\cdot g - \int f'\cdot g = x^2\ln(x) - \int \frac{1}{\cancel{x}}\cdot x^{\cancel{2}}\,dx = \boxed{x^2\ln(x) - \frac{x^2}{2} + c}$$

$g = x^2$ $f' = 1/x$

E389

$$\int \underbrace{x}_{f}\,\underbrace{e^x}_{g'}\,dx = xe^x - \int e^x\,dx = xe^x - e^x + c = \boxed{e^x(x-1)+c}$$

$f' = 1$ $g = e^x$

E393

$(1-x^2)^{-1/2}$ $(-2x)$

$$\int \arcsin(x)\,dx = \int \underbrace{\arcsin(x)}_{f}\cdot\underbrace{x^0}_{g'}\,dx = x\arcsin(x) - \int \frac{-2x}{\sqrt{1-x^2}}\left(-\frac{1}{2}\right)dx =$$

$f' = \dfrac{1}{\sqrt{1-x^2}}$ $g = x$

$$= x\arcsin(x) - \sqrt{1-x^2}\,\cancel{2}\cdot\left(-\frac{1}{\cancel{2}}\right) + c$$

$$= \boxed{x\arcsin(x) + \sqrt{1-x^2} + c}$$

In classe: Integrazione per Parti

$$\int f g' = f g - \int f' g$$

E399 P1295

$$\int \underbrace{\sqrt[3]{x}}_{g'} \; \underbrace{\ln(2x)}_{f} \, dx = \ln(2x) \cdot \frac{3}{4} x \sqrt[3]{x} - \int x^{-1} \cdot \frac{3}{4} x^{4/3}$$

$g = x^{4/3} \cdot \frac{3}{4}$ $\quad f' = \frac{1}{\cancel{2}x} \cdot \cancel{2}$

$g = \frac{3}{4} x \sqrt[3]{x}$ $\quad f' = \frac{1}{x}$

$$= \ln(2x) \cdot \frac{3}{4} x \sqrt[3]{x} - \int \frac{3}{4} x^{1/3}$$

$$= \ln(2x) \cdot \frac{3}{4} x \sqrt[3]{x} - \frac{3}{4} x^{4/3} \cdot \frac{3}{4}$$

$$= \boxed{\frac{3}{4} x^{4/3} \left(\ln(2x) - \frac{3}{4} \right) + c}$$

E406

$$\int \frac{x}{2\sqrt{x+1}} \, dx = \frac{1}{2} \int \underbrace{x}_{f} \cdot \underbrace{(x+1)^{-1/2}}_{g'} \, dx = \frac{1}{\cancel{2}} \left(x \cdot (x+1)^{1/2} \cdot \cancel{2} \right) - \overset{1/2}{\int} 2(x+1)^{1/2} =$$

$f' = 1$ $\quad g = (x+1)^{1/2} \cdot 2$

$$\cancel{1/2} \cdot \cancel{2}$$

$$= x(x+1)^{1/2} - \cancel{2}(x+1)^{3/2} \cdot \frac{2}{3} =$$

$$= x(x+1)^{1/2} - \frac{2}{3}(x+1)^{3/2} =$$

$$= x(x+1)^{1/2} - \frac{2}{3}(x+1)(x+1)^{1/2} =$$

$$= (x+1)^{1/2} \left(x - \frac{2}{3} x - \frac{2}{3} \right) =$$

$$= \boxed{(x+1)^{1/2} \left(\frac{1}{3} x - \frac{2}{3} \right) + c}$$

E420

$$\int \underbrace{\frac{x}{\sqrt{1-x^2}}}_{g'} \underbrace{\arcsin(x)}_{f} \, dx = \arcsin(x) \cdot -(1-x^2)^{1/2} - \int \cancel{\frac{1}{\sqrt{1-x^2}}} \cdot (-\cancel{(1-x^2)})^{1/2}$$

$$f' = \frac{1}{\sqrt{1-x^2}}$$

$$= \arcsin(x) - (1-x^2)^{1/2} - \int -1$$

$$= \boxed{\arcsin(x) - (1-x^2)^{1/2} + x + c}$$

$$g = -\frac{1}{2} \cdot -2x \cdot \frac{1}{\sqrt{1-x^2}}$$

$$g = -\frac{1}{\cancel{2}} (1-x^2)^{1/2} \cdot \cancel{2}$$

$$g = -(1-x^2)^{1/2}$$

E423

$$\int \underbrace{e^x}_{f} \underbrace{\cos(x)}_{g'} \, dx = e^x \sin(x) - \int \underbrace{e^x}_{f} \underbrace{\sin(x)}_{g'}$$

$$f' = e^x \qquad g = \sin(x) \qquad f' = e^x \qquad g = -\cos(x)$$

$$\int e^x \cos(x) \, dx = e^x \sin(x) - \left[-e^x \cos(x) - \int -e^x \cos(x) \, dx \right]$$

$$\int e^x \cos(x) \, dx = e^x \sin(x) + e^x \cos(x) - \int e^x \cos(x) \, dx$$

$$\frac{\cancel{2} \int e^x \cos(x) \, dx}{\cancel{2}} = \frac{e^x \sin(x) + e^x \cos(x)}{2}$$

$$\int e^x \cos(x) \, dx = \boxed{\frac{e^x \sin(x) + e^x \cos(x)}{2} + c}$$

Compito 18/11/2021

~~411~~ - ~~413~~ - ~~416~~ - 421 - ~~422~~ - ~~424~~

E411 P1295

$$\int \underset{g'}{(2x+1)} \underset{f}{\ln(x+1)}\, dx = \ln(x+1)\, x(x+1) - \int \frac{x\cancel{(x+1)}}{\cancel{x+1}}\, dx =$$

$g = x^2 + x \quad f' = \frac{1}{x+1}$

$g = x(x+1)$

$$= \boxed{x(x+1)\ln(x+1) - \frac{x^2}{2} + c}$$

E413

$$\int \underset{f}{x^2} \underset{g'}{e^x}\, dx = x^2 e^x - \int \underset{f}{2x} \underset{g'}{e^x}\, dx = x^2 e^x - \left(2x e^x - \int 2 e^x\, dx\right) =$$

$f' = 2x \quad g = e^x \qquad f' = 2 \quad g = e^x$

$$= x^2 e^x - 2x e^x - 2e^x + c$$

$$= \boxed{e^x(x^2 - 2x - 2) + c}$$

E416

$$\int \frac{x}{\cos^2(x)}\, dx = \int \underset{f}{x} \cdot \underset{g'}{\frac{1}{\cos^2(x)}}\, dx = x\tan(x) - \int \frac{\overset{f'(x)}{\sin(x)}}{\underset{f(x)}{\cos(x)}}\, dx =$$

$f' = 1 \quad g = \tan(x)$

$$= \boxed{x\tan(x) - \ln|\cos(x)| + c}$$

E421

$$\int \underset{g'}{x} \underset{f}{\arcsin\left(\frac{1}{x}\right)}\, dx = \frac{x^2}{2}\arcsin\left(\frac{1}{x}\right) - \int \frac{\cancel{x^2}}{2}\left(-\frac{1}{\cancel{x^2}}\right)\frac{1}{\sqrt{1 - \frac{1}{x^2}}}\, dx$$

$g = \frac{x^2}{2} \quad f' = \frac{1}{\sqrt{1 - \frac{1}{x^2}}} \cdot \left(-\frac{1}{x^2}\right)$

$$\sqrt{\frac{x^2-1}{x^2}} = \frac{\sqrt{x^2-1}}{|x|}$$

$$= \frac{x^2}{2}\arcsin\left(\frac{1}{x}\right) + \frac{1}{2}\int \frac{1}{\sqrt{1 - \frac{1}{x^2}}}\, dx$$

$$= \frac{x^2}{2}\arcsin\left(\frac{1}{x}\right) + \frac{1}{2}\int \frac{1}{2} \cdot \frac{2|x|}{\sqrt{x^2-1}}\, dx$$

$$\boxed{|x| = \text{sign}(x) = \frac{x}{|x|}}$$

"+1" se $x \geq 0$

"-1" se $x < 0$

segno

$$= \frac{x^2}{2}\arcsin\left(\frac{1}{x}\right) + \frac{1}{4}\frac{|x|}{x}\int \frac{2x}{\sqrt{x^2-1}}\, dx \qquad (x^2-1)^{-1/2} \cdot 2x$$

$$= \frac{x^2}{2} \arcsin\left(\frac{1}{x}\right) + \frac{1}{\cancel{4}\,2} \frac{|x|}{x} \cdot \cancel{2}\sqrt{x^2-1} + c$$

$$\boxed{= \frac{x^2}{2} \arcsin\left(\frac{1}{x}\right) + \frac{1}{2} \frac{|x|}{x} \cdot \sqrt{x^2-1} + c}$$

$D\left(e^{f(x)}\right) = e^{f(x)} \cdot f'(x)$
↳ "-1"

E424

$$\int \frac{\overset{f}{\cos(x)}}{\underset{g'}{e^{x+1}}} \, dx = -e^{-x-1} \cos(x) - \int -e^{-x-1} (-\sin(x)) \, dx$$

$f' = -\sin(x)$
$g = -e^{-x-1}$

$$= -e^{-x-1} \cos(x) - \int \underbrace{e^{-x-1}}_{g'} \underbrace{\sin(x)}_{f} \, dx$$

$g = -e^{-x-1}$ $\quad f' = \cos(x)$

$$= -e^{-x-1} \cos(x) - \left(-e^{-x-1} \sin(x) - \int \cos(x) \cdot (-e^{-x-1}) \, dx \right.$$

$$= -e^{-x-1} \cos(x) + e^{-x-1} \sin(x) + \int -e^{-x-1} \cos(x) \, dx$$

$$\int e^{-x-1} \cos(x) = -e^{-x-1} \cos(x) + e^{-x-1} \sin(x) - \int e^{-x-1} \cos(x) \, dx$$

$$2 \int \frac{\cos(x)}{e^{x+1}} \, dx = e^{-x-1} (\sin(x) - \cos(x))$$

$$\int \frac{\cos(x)}{e^{x+1}} \, dx = \boxed{\frac{e^{-x-1} (\sin(x) - \cos(x))}{2} + C}$$

Compito 20/11/21

~~414~~ - ~~415~~ - ~~425~~ - ~~426~~

E414 P1295

Oppure sommo "+1" e "-1"

$$\int \ln(2x+1)\,dx = \int \underbrace{\ln(2x+1)}_{f} \cdot \underbrace{x^0}_{g'}\,dx = \ln(2x+1)\,x - \int \underbrace{\frac{2x}{2x+1}}\,dx$$

$f' = \frac{2}{2x+1}$ $\qquad$ $g = x$

sostit: $2x+1 = t$

$dt = 2\,dx$

$dx = dt \cdot \frac{1}{2}$

$2x = t-1$

$$= \ln(2x+1)\,x - \int \frac{t-1}{t} \cdot \frac{1}{2}\,dt =$$

$$= x\ln(2x+1) - \frac{1}{2}\int dt - \frac{1}{2}\int -\frac{1}{t}\,dt =$$

$$= x\ln(2x+1) - \frac{1}{2}t + \frac{1}{2}\ln|t| + c =$$

$$= x\ln(2x+1) - \frac{1}{2}(2x+1) + \frac{1}{2}\ln(2x+1) + c$$

$$= \ln(2x+1)\left(x+\frac{1}{2}\right) - x - \frac{1}{2} + c =$$

$$= \boxed{\ln(2x+1)\left(x+\frac{1}{2}\right) - x + c}$$

E415

$$\int \underbrace{(x)^{1/2}}_{g'} \underbrace{\ln(x)}_{f} \, dx = \frac{2}{3} x\sqrt{x} \ln(x) - \int \frac{1}{\cancel{x}} \cdot \frac{2}{3} \cancel{x}\sqrt{x} \, dx$$

$g = \frac{2}{3} x^{3/2}$ $\quad f' = \frac{1}{x}$

$g = \frac{2}{3} x\sqrt{x}$

$$= \frac{2}{3} x\sqrt{x} \ln(x) - \int \frac{2}{3} (x)^{1/2} \, dx$$

$$= \frac{2}{3} x\sqrt{x} \ln(x) - \frac{2}{3} x^{3/2} \frac{2}{3} + c$$

$$= \frac{2}{3} x\sqrt{x} \ln(x) - \frac{4}{9} x\sqrt{x} + c$$

$$= \boxed{\frac{2}{3} x\sqrt{x} \left(\ln(x) - \frac{2}{3} \right) + c}$$

E425

$$\int \underbrace{e^{2x}}_{g'} \cdot \underbrace{\sin(x)}_{f} \, dx = \frac{e^{2x}}{2} \sin(x) - \int \frac{e^{2x}}{2} \cos(x) \, dx$$

$g = \frac{1}{2} \cdot 2e^{2x}$ $\quad f' = \cos(x)$

$g = \frac{e^{2x}}{2}$

$$= \frac{e^{2x}}{2} \sin(x) - \frac{1}{2} \int \underbrace{e^{2x}}_{g'} \underbrace{\cos(x)}_{f} \, dx$$

$g = \frac{1}{2} 2e^{2x}$ $\quad f' = -\sin(x)$

$g = \frac{e^{2x}}{2}$

$\frac{1}{2} + 1 = \frac{3}{2}$

$$= \frac{e^{2x}}{2} \sin(x) - \frac{1}{2} \left(\frac{e^{2x}}{2} \cos(x) - \int -\frac{e^{2x}}{2} \cdot \sin(x) \, dx \right)$$

$$\int e^{2x} \cdot \sin(x) \, dx = \frac{e^{2x}}{2} \sin(x) - \frac{e^{2x}}{4} \cos(x) - \frac{1}{2} \int e^{2x} \sin(x) \, dx$$

$$\frac{3}{2} \int e^{2x} \sin(x) \, dx = \frac{e^{2x}}{2} \left(\sin(x) - \frac{\cos(x)}{2} \right)$$

$$\int e^{2x} \sin(x) \, dx = \frac{2e^{2x}}{5} \left(\sin(x) - \frac{\cos(x)}{2} \right) + c$$

$$\int e^{2x} \sin(x) \, dx = \boxed{\frac{e^{2x}}{5} \left(2\sin(x) - \cos(x) \right) + c}$$

E426

$$\int \frac{\sin(x)}{e^x}\,dx =$$

$$\int \underbrace{e^{-x}}_{g'} \underbrace{\sin(x)}_{f}\,dx = -e^{-x}\sin(x) - \int -e^{-x}\cos(x)\,dx$$

$g = -1 \cdot -1 e^{-x}$ $\quad f' = \cos(x)$

$g = -e^{-x}$

$$= -e^{-x}\sin(x) + \int \underbrace{e^{-x}}_{g'} \underbrace{\cos(x)}_{f}\,dx$$

$g = -1 \cdot -1 e^{-x}$ $\quad f' = -\sin(x)$

$g = -e^{-x}$

$$= -e^{-x}\sin(x) + \left(-e^{-x}\cos(x) - \int -e^{-x}(-\sin(x))\,dx\right)$$

$$\int e^{-x}\sin(x)\,dx = -e^{-x}\sin(x) - e^{-x}\cos(x) - \int e^{-x}\sin(x)\,dx$$

$$2\int e^{-x}\sin(x)\,dx = -e^{-x}(\sin(x)+\cos(x))$$

$$\int e^{-x}\sin(x)\,dx = \frac{-e^{-x}}{2}(\sin(x)+\cos(x)) + C$$

$$= \boxed{-\frac{\sin(x)+\cos(x)}{2e^x} + C}$$

Compiti 23/11/21

~~435~~ - ~~438~~ - ~~440~~ - ~~441~~ - ~~442~~

E435 P1297

$$\int \frac{\overset{f'(x)}{2x+1}}{\underset{f(x)}{x^2+x}}\,dx = \boxed{\ln|x^2+x| + c}$$

E438

$$\int \frac{2x^9+x^4+1}{\underset{f(x)}{x^{10}+x^5+5x}}\,dx = \int \frac{1}{5}\cdot 5\,\frac{2x^9+x^4+1}{x^{10}+x^5+5x}\,dx = \boxed{\frac{1}{5}\ln|x^{10}+x^5+5x| + c}$$

$f'(x) = 10x^9+5x^4+5$

$f'(x) = 5(2x^2+x^4+1)$

E440

$$\int \frac{x^2-1}{\underset{f(x)}{x^3-3x+1}}\,dx = \int \frac{1}{3}\cdot 3\,\frac{x^2-1}{x^3-3x+1}\,dx = \boxed{\frac{1}{3}\ln|x^3-3x+1|}$$

$f'(x) = 3x^2-3$

$f'(x) = 3(x^2-1)$

E441

$$\int \frac{4x^2+10x+3}{2x^2+3x}\,dx =$$

$$\begin{array}{l|l} 4x^2+10x+3 & 2x^2+3x \\ \hline -4x^2-6x & 2 \\ \hline \diagup \quad 4x+3 & \end{array}$$

$$\int 2\,dx + \int \frac{4x+3}{\underset{f(x)}{2x^2+3x}}\,dx = \boxed{2x + \ln|4x+3| + c}$$

$f'(x) = 4x+3$

E442

$$\int \frac{2x^3+x^2+1}{x^2+1}\,dx$$

$$\begin{array}{l|l} 2x^3+x^2+0x+1 & x^2+1 \\ \hline -2x^3 \qquad -2x & 2x+1 \\ \hline \diagup \quad x^2-2x+1 & \\ \quad -x^2 \qquad\quad -1 & \\ \hline \quad \diagup \quad -2x & \end{array}$$

$$\int 2x+1\,dx + \int \frac{-2x}{x^2+1}\,dx$$

$f(x)$

$f'(x) = 2x$

$$\int 2x+1\,dx + \int -1 \cdot \frac{-1(-2x)}{x^2+1}\,dx = \boxed{x^2+x-\ln|x^2+1|+c}$$

Compito 25/11/21

E 466 - ~~467~~ - ~~468~~

E466 P1299

$$\int \frac{-5}{3x^2 - x - 2}\, dx$$

calcolo il delta : $\Delta = 1 + 24 = 25$

$$x_{1,2} = \frac{1 \pm 5}{6} \quad \begin{cases} x_1 = 1 \\ x_2 = -2/3 \end{cases}$$

$$3x^2 - x - 2 = a(x - x_1)(x - x_2)$$

$$\frac{-5}{3(x-1)(x+2/3)} = \frac{A}{3(x-1)} + \frac{B}{(x+2/3)} = \frac{A(x+2/3) + 3B(x-1)}{3(x-1)(x+2/3)}$$

$$= \frac{x(A+3B) + 2/3\,A - 3B}{3(x-1)(x+2/3)}$$

$$\begin{cases} A + 3B = 0 \\ 2/3\,A - 3B = -5 \end{cases} \quad \begin{cases} A = -3B \\ +2B + 3B = +5 \end{cases} \quad \begin{cases} A = -3B \\ 5B = 5 \end{cases} \quad \begin{cases} A = -3B \\ B = 1 \end{cases}$$

$$\begin{cases} A = -3 \\ B = 1 \end{cases} \Rightarrow \frac{-3}{3(x-1)} + \frac{1}{(x+2/3)}$$

$$\int \frac{-\cancel{3}}{\cancel{3}(x-1)}\, dx + \int \frac{1}{(x+2/3)}\, dx$$

$$\int \frac{-1}{(x-1)}\, dx + \int \frac{1}{(x+2/3)}\, dx$$

$$= -\ln(x-1) + 3\ln(3x+2)$$

$$= \ln\left|\frac{3x+2}{3(x-1)}\right| + K$$

$\ln(A/B) = \ln(A) - \ln(B)$

$$= \ln\left|\frac{3x+2}{x-1}\right| \underbrace{- \ln(3) + K}_{K} \Rightarrow \boxed{\ln\left|\frac{3x+2}{x-1}\right| + K}$$

E467

$$\int \frac{9-9x}{9x^2-1}\,dx$$

delta : $\Delta = 36$

$$x_{12} = \frac{0 \pm 6}{18} \quad \begin{cases} x_1 = 1/3 \\ x_2 = -1/3 \end{cases}$$

$$\boxed{\text{scompongo : } \quad 9x^2-1 = 9\left(x-1/3\right)\left(x+1/3\right)}$$

$$\frac{9-9x}{9\left(x-1/3\right)\left(x+1/3\right)} = \frac{A}{9\left(x-1/3\right)} + \frac{B}{\left(x+1/3\right)}$$

$$= \frac{A\left(x+1/3\right) + 9B\left(x-1/3\right)}{9\left(x-1/3\right)\left(x+1/3\right)} = \frac{x\left(A+9B\right) + 1/3\,A - 3B}{9\left(x-1/3\right)\left(x+1/3\right)}$$

$$\begin{cases} A+9B = -9 \\ 1/3\,A - 3B = 9 \end{cases} \quad \begin{cases} A = -9-9B \\ -3-3B-3B = 9 \end{cases} \quad \begin{cases} \text{"} \\ -6B = 12 \end{cases} \quad \begin{cases} \text{"} \\ B = -2 \end{cases}$$

$$\begin{cases} A = -9-(-18) \\ B = -2 \end{cases} \quad \begin{cases} A = 9 \\ B = -2 \end{cases}$$

$$\int \left(\frac{\cancel{9}}{\cancel{9}\left(x-1/3\right)} + \frac{-2}{\left(x+1/3\right)} \right) dx$$

$$\int \left(\frac{1}{\frac{1}{3}\left(3x-1\right)} + \frac{-2}{\frac{1}{3}\left(3x+1\right)} \right) dx$$

$$\int \left(\frac{3}{3x-1} + \frac{-6}{3x+1} \right) dx$$

$$= \ln|3x-1| - 2\ln|3x+1|$$

$$= \boxed{\ln\left(\frac{|3x-1|}{\left(|3x+1|\right)^2}\right) + c}$$

E468

$$\int \frac{x^4 + x^3 + 6}{x^2 + x}\, dx$$

$$\begin{array}{r|l} x^4 + x^3 + 6 & x^2 + x \\ \hline -x^4 - x^3 & x^2 \\ \hline 6 & \end{array}$$

$$\int x^2\, dx + \boxed{\int \frac{6}{x^2 + x}\, dx}$$

$\Delta = 1$

$$x_{1,2} = \frac{-1 \pm 1}{2} \quad \begin{cases} 0 & (x_1) \\ -1 & (x_2) \end{cases}$$

$$x^2 + x = (x)(x+1)$$

$$\frac{6}{x^2 + x} = \frac{A}{x} + \frac{B}{x+1} = \frac{A(x+1) + B(x)}{(x)(x+1)}$$

$$= \frac{x(A+B) + A}{(x)(x+1)}$$

$$\begin{cases} A + B = 0 \\ A = 6 \end{cases} \quad \begin{cases} B = -A \\ A = 6 \end{cases} \quad \begin{cases} B = -6 \\ A = 6 \end{cases}$$

$$\frac{6}{x} + \frac{-6}{x+1}$$

$$\int x^2\, dx + \boxed{\int \frac{6}{x}\, dx + \int \frac{-6}{x+1}\, dx}$$

$$= \frac{x^3}{3} + 6 \ln|x| - 6 \ln|x+1| + c$$

$$= \frac{x^3}{3} + \ln\left(\frac{(|x|)^6}{(|x+1|)^6}\right) + c$$

$$= \frac{x^3}{3} + \ln\left(\frac{|x|}{|x+1|}\right)^6 + c$$

$$\boxed{= \frac{x^3}{3} + 6 \ln\left(\frac{|x|}{|x+1|}\right) + c}$$

In classe

E 477 P 1299

$$\int \frac{x^2 - x + 1}{x^2 - 2x + 1}$$

$$\begin{array}{l|l} x^2 - x + 1 & x^2 - 2x + 1 \\ \hline -x^2 + 2x - 1 & 1 \\ \hline / \quad x \quad / & \end{array}$$

$\Delta = 0$

N grado 1

$$\int 1\,dx + \int \frac{x}{x^2 - 2x + 1}\,dx$$

$$= x + \int \frac{x}{(x-1)^2}\,dx$$

$$\frac{x}{(x-1)^2} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2}$$

$$\frac{x}{(x-1)^2} = \frac{A(x-1) + B}{(x-1)^2}$$

$$\begin{cases} A = 1 \\ -A + B = 0 \end{cases} \qquad \begin{cases} A = 1 \\ B = 1 \end{cases}$$

$$= x + \int \frac{1}{x-1}\,dx + \int \frac{1}{(x-1)^2}\,dx$$

$$= x + \ln|x-1| + \frac{(x-1)^{-1}}{-1} + c$$

$$= \boxed{x + \ln|x-1| - \frac{1}{(x-1)} + c}$$

Compito 27/11/2021

~~470~~ - ~~472~~ - ~~473~~ - ~~474~~ P1299

E470 P1299

$$\int \frac{2}{x^2-6x+9}\,dx$$

$$\int \frac{2}{1(x-3)^2}\,dx$$

$$2\int \frac{1}{(x-3)^2}\,dx = 2\int (x-3)^{-2}(1)\,dx = \frac{2(x-3)^{-1}}{-1} + c = \boxed{\frac{-2}{x-3} + c}$$

E472

$$\int \frac{x}{x^2-4x+4}\,dx$$

$$\int \frac{x}{1(x-2)^2}\,dx = \int \frac{A}{1(x-2)}\,dx + \int \frac{B}{(x-2)^2}\,dx$$

$$\frac{x}{\cancel{(x-2)^2}} = \frac{Ax-2A+B}{\cancel{(x-2)^2}}$$

$$x = Ax - 2A + B \qquad \begin{cases} A = 1 \\ -2A+B = 0 \end{cases} \qquad \begin{cases} A = 1 \\ B = 2 \end{cases}$$

$$\int \frac{1}{1(x-2)}\,dx + 2\int \frac{1}{\overbrace{(x-2)^2}^{\Delta=0}}\,dx =$$

$$= \ln|x-2| + \frac{2(x-2)^{-1}}{-1} + c$$

$$= \boxed{\ln|x-2| - \frac{2}{(x-2)} + c}$$

E473

$$\int \frac{2x-1}{x^2+2x+1}\,dx$$

$$\int \frac{2x-1}{(x+1)^2}\,dx \Rightarrow \frac{A}{(x+1)} + \frac{B}{(x+1)^2}$$

$$2x-1 = Ax + A + B$$

$$A = 2 \qquad\qquad A = 2$$

$$B = -1 - A \qquad\qquad B = -3$$

$$2\int \frac{1}{(x+1)}\,dx - 3\overbrace{\int \frac{1}{(x+1)^2}\,dx}^{\Delta=0}$$

$$= \boxed{2\ln|x+1| + \frac{3}{(x+1)} + c}$$

E474

$$\int \frac{4x+1}{4x^2+4x+1}\,dx$$

$$\int \frac{4x+1}{4(x+1/2)^2}\,dx \Rightarrow \frac{A}{4(x+1/2)} + \frac{B}{(x+1/2)^2}$$

$$\frac{4x+1}{\cancel{4(x+1/2)^2}} = \frac{Ax + 1/2A + 4B}{\cancel{4(x+1/2)^2}}$$

$$A = 4 \qquad\qquad A = 4$$

$$B = \frac{1-2}{4} \qquad\qquad B = -1/4$$

$$\int \frac{\cancel{4}\,2}{2\,\cancel{4}\left(\frac{2x+1}{\cancel{2}}\right)}\,dx + \int -\frac{1}{4} \cdot \frac{2^{2}}{(2x+1)^2}\,dx$$

ricorda che c'è l'esponente

$$= \ln|2x+1| - \frac{1}{2\,\cancel{4}} \cdot \cancel{2} \cdot \frac{(2x+1)^{-1}}{-1}$$

$$= \ln|2x+1| + \frac{1}{2} \cdot \frac{1}{2x+1} + c$$

$$= \boxed{\ln|2x+1| + \frac{1}{4x+2} + c}$$

In classe

E491 P1301

$$\int \frac{x+3}{x^2+4x+5}\,dx$$

$$\boxed{\Delta<0}$$

$f' = 2x+4$

$$\frac{1}{2}\left(\int \frac{x+4}{x^2+4x+5} + \frac{2}{x^2+4x+5}\,dx\right)$$

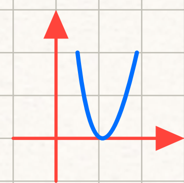
$$\frac{1}{2}\int \frac{x+4}{x^2+4x+5}\,dx + \frac{1}{\cancel{2}}\int \frac{\cancel{2}}{x^2+4x+5}\,dx$$

$$= \frac{1}{2}\ln(x^2+4x+5) + \int \frac{1}{1+(x+2)^2}\,dx$$

$$= \frac{1}{2}\ln(x^2+4x+5) + \arctan(x+2) + K$$

posso non mettere il modulo perché ha il $\Delta<0$.

se la "$a \geq 0$" e "$\Delta<0$" $\rightarrow$ sempre positivo



se la "$a<0$" e "$\Delta<0$" $\rightarrow$ sempre negativo